Math 382 **2nd Semester 2016-2017**
Mid-Exam 2 **4/5/2017**
Duration: 90 Minutes

Question 1:

i) State the definition of continuous function at a point c. [Definition 6.1]
ii) State the definition of a right hand limit of a function f at a point c. [Definition 5.2]
iii) Give an example of two functions one of which is not continuous at c but the composition is continuous at c.

Question 2:

i) Let $f: D \to \mathbb{R}$ and $c \in \widehat{D}$. Prove that if for any sequence (x_n) in D, $x_n \neq c$ and $x_n \to c$ satisfy $f(x_{n)} \to l$, **then** $\lim_{x \to c} f(x) = l$. [Theorem 5.1].
ii) Let $f: \mathbb{R} \to \mathbb{R}$ satisfy the relation $f(x+y) = f(x) + f(y)$ for all x, y. If $\lim_{x \to c} f(x) = l$ for some $c \in \mathbb{R}$. **Prove that f has a limit at every point in** $\mathbb{R}$. [Exercise 9 in section 5.2]

Question 3:

i) Prove that $\lim_{x \to 0} sin(x) = 0$. [Example 5.8]
ii) Let $f: D \to \mathbb{R}$, c an isolated point in D. **Show that f is continuous at c.** [Note after Definition 6.1]
iii) Prove that if a function f is continuous at a point c, then so is the function $\sqrt{f(x)}$. [Example 6.11].

Question 4:

a) Find the limit if it exists. Write all the details

i) $\lim_{x \to \infty} cos(\pi x)$.
ii) $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} x, & x \in \mathbb{Q} \\ -x, & x \notin \mathbb{Q} \end{cases}$. [exercise 5 in section 5.1]

b) Explain why we cannot find two functions $f, g: D \to \mathbb{R}$ for which f is continuous at c but g is not continuous at c. **But the sum $\boldsymbol{f + g}$ is continuous at $\boldsymbol{c}$.**

c) Explain why we cannot find two functions both are continuous at c, but the **composition is not continuous at c.**