<u>Question 1</u>: [6.5 Marks] Let $\mathcal{R} = \{(a, \infty): a \in \mathbb{R}\} \cup \{\mathbb{R}, \emptyset\}$.

a) Prove that $\mathcal{R}$ is a topology on $\mathbb{R}$. [3.5]

b) Find all closed subsets of $(\mathbb{R}, \mathcal{R})$. [1]

c) Find the closure of the subsets:$(1, \infty)$ and $(1,2)$. [1]

d) Find A', where $A = (0,1)$.[1]

<u>Question 2:</u> [6.5 Marks]

a) Show that $\mathcal{B} = \{(-\infty, a) : a \in \mathbb{Q}\}$ is a base for the left ray topology $\mathcal{L}$ on $\mathbb{R}$.[1.5]

b) Find the topology generated by the collection $\mathcal{S} = \{\{a\}, \{b, c\}, \{a, c\}\}$ on $X = \{a, b, c\}$.[2]

c) Let $A = (0,1) \cup \{2\}$ and $B = \mathbb{Q} \cap [0,1]$ be subsets of $(\mathbb{R}, \mathcal{U})$. Find $Int(A), Int(B), Bd(A), Bd(B), Ext(A)$, and $Ext(B)$.[3]

<u>Question 3:</u> [7.5 Marks]

a) Let $(X, \mathcal{T})$ be a topological space and $A \subseteq X$. Show that

$$\bar{A} = A \cup A'. \qquad [2]$$

b) If A and B are subsets of a topological space $(X, \mathcal{T})$. Show that

$$\overline{A \cap B} \subseteq \bar{A} \cap \bar{B} \qquad [1.5]$$

c) Give an example of each of the following:

1. A topological space X and a proper dense subset of it. [1]

2. A topological space X and subsets A and B of X with

$$\mathrm{Int}(A) \cup \mathrm{Int}(B) \neq \mathrm{Int}(A \cup B). \qquad [1]$$

3. A topological space X and a subset A of X which is neither open nor closed.[1]

4. A topological space X and a subset A of X where $\mathrm{Bd}(A) = \emptyset$. [1]

<u>Question 4:</u> [4.5 Marks]

Prove or disprove each of the following

a) The collection $\{X, \emptyset, (0,1), \{1\}, (0,1]\}$ is a topology on $X = [0,1]$.

b) The usual topology $\mathcal{U}$ is finer than the cofinite topology $\mathcal{T}_{cof}$ on $\mathbb{R}$.

c) $\mathrm{Bd}(A)$ is always closed, where A is a subset of a topological space X.