Question 1: [7.5 Marks] Let $\mathcal{L} = \{\{(-\infty, a): a \in \mathbb{R}\} \cup \{\mathbb{R}, \emptyset\}$.

a) Prove that $\mathcal{L}$ is a topology on $\mathbb{R}$. [3.5]

b) Find all closed subsets in $(\mathbb{R}, \mathcal{L})$. [1.5]

c) Find the closure of the subsets: $(1, \infty), (-\infty, 2], (0,1)$. [1.5]

d) Find A', where $A = (0,1)$.[1]

Question 2: [7 Marks]

a) Show that $\mathcal{B} = \{(a, b): a, b \in \mathbb{Q}\}$ is a base for $\mathcal{U}$.[2]

b) Find the topology generated by the collection $\mathcal{S} = \{\{a, b\}, \{b, c\}, \{a, c\}\}$ on $X = \{a, b, c\}$.[2]

c) Let $A = (0,1] \subseteq (\mathbb{R}, \mathcal{U})$ and $B = \{0,1\} \subseteq (\mathbb{R}, \tau_{cof})$. Find $Int(A), Int(B), Bd(A), Bd(B), Ext(A)$, and $Ext(B)$.[3]

<u>Question 3:</u> [6 Marks]

a) If U is open subset of $(\mathbb{R}, \mathcal{U})$, show that U is a union of open intervals.[2]

b) If A is a closed subset of a topological space $(X, \mathcal{T})$, show that $A' \subseteq A$.[1.5]

c) Let $\{U_\alpha : \alpha \in \Delta\}$ be a collection of closed subsets in a topological space $(X, \mathcal{T})$.

1. show that $\cap \{U_\alpha : \alpha \in \Delta\}$ is closed. [1.5]

2. Give an example to show that $\cup \{U_\alpha : \alpha \in \Delta\}$ need not be closed.[1]

<u>Question 4:</u> [4.5 Marks]

Prove or disprove each of the following

a) The collection $\{X, \emptyset, \{a\}, \{b, c\}\}$ is a topology on $X = \{a, b, c, d\}$.

b) The open half line topology $\mathcal{C}$ is finer than the usual topology $\mathcal{U}$ on $\mathbb{R}$.

c) $\mathbb{Q}$ is dense in $\mathbb{R}$.