King Saud University | Time allowed: 3 hours

Mathematics Department | Semester: 1447-1

# MATH 373 - Final Examination

**Question 1.** [8] Let $\mathcal{T}_{cof} = \{U \subseteq \mathbb{R} \mid \mathbb{R} - U \text{ is finite}\} \cup \{\emptyset\}$.

1. Show that $\mathcal{T}_{cof}$ is a topology on $\mathbb{R}$. [3]
2. Describe the closed subsets of $(\mathbb{R}, \mathcal{T}_{cof})$. [2]
3. Is $(\mathbb{R}, \mathcal{T}_{cof})$ Hausdorff? Justify your answer. [2]
4. Is $(\mathbb{R}, \mathcal{T}_{cof})$ metrizable? Justify your answer. [1]

**Question 2.** [6=6x1] Consider $\mathbb{R}$ with the half-closed interval topology $\mathcal{H}$ and let $A = \{0,1\}$ and $B = [0,1]$. Find $Cl(A)$, $Cl(B)$, $Int(A)$, $Int(B)$, $A'$, and $B'$.

**Question 3.** [6]

1. Give the definition of a continuous function, an open function, a closed function, and a homeomorphism. [2]
2. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} 1, & x > 2 \\ x, & x \leq 2 \end{cases}$
   i. Is $f$ $\mathcal{U} - \mathcal{U}$ continuous? Why. [2]
   ii. Is $f$ $\mathcal{U} - \mathcal{U}$ open? Why. [2]

**Question 4.** [6]

1. Show that having a proper nonempty subset that is open and closed is a topological property. [2]
2. Show that $(\mathbb{R}, \mathcal{U})$ is not compact. [2]
3. Let $X = \{a, b, c\}$ with the topology $\mathcal{T} = \{X, \emptyset, \{a, b\}, \{c\}\}$. Find a base for the product topology on $X \times X$. [2]

**Question 5.** [6] Let $d$ be the usual metric for $\mathbb{R}$ given by $d(x, y) = |x - y|$ and define $e: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by

$$e(x, y) = \min\{d(x, y), 1\}$$

1. Show that $e$ is a metric for $\mathbb{R}$. [3]
2. Find $B_1^e(1)$ and $B_3^e(2)$. [2]
3. Is $e$ bounded? Why? [1]

**Question 6.** [8]

1. Explain what it means for a topological space to be sequentially compact and to be limit-point compact. [2]
2. Show that every sequentially compact space is limit point compact. [3]
3. Let $\mathcal{B} = \{\{1,2\}, \{3,4\}, \{5,6\}, \dots, \{n, n+1\}, \dots\}$ and let $\mathcal{T}(\mathcal{B})$ be the topology on $\mathbb{N}$ generated by $\mathcal{B}$.
   i. Show that $(\mathbb{N}, \mathcal{T}(\mathcal{B}))$ is not compact. [1.5]
   ii. Show that $(\mathbb{N}, \mathcal{T}(\mathcal{B}))$ is limit-point compact. [1.5]