King Saud University – Department of Mathematics
Math 373 – First Midterm Exam (1447-1)
Duration: 1 h 30 mn (Total: 25 points)

Instructions: Answer all questions. Provide clear reasoning and proofs for each result.

1. (6 points) Let $X = \mathbb{R}$, and define

$$\mathcal{L} = \{\mathbb{R}, \emptyset\} \cup \{(-\infty, a),\ a \in \mathbb{R}\}.$$

   1. Show that $\mathcal{L}$ is a topology on X (called the left ray topology). (3 points)
   2. Identify the closed subsets of X. (1 point)
   3. Identify the closure of the subset $\mathbb{Z}$. (1 point)
   4. Give two different dense subsets of X. (1 point)

2. (6 points) In the space $X = \mathbb{R}$ with the usual topology $\mathcal{U}$, determine the derived set, interior, exterior, and boundary of each of the following sets:

   1. $A = [0, 1)$, (2 points)
   2. $B = \mathbb{Q}$, (2 points)
   3. $C = [0, 1] \cup \{2\}$. (2 points)

3. (5 points) Consider the collection $\mathcal{B} = \{(a, b) : a, b \in \mathbb{Q}, a < b\}$ of all open intervals with rational endpoints in $\mathbb{R}$.

   1. Show that $\mathcal{B}$ is a basis for some topology $\mathcal{T}$ on $\mathbb{R}$. (2.5 points)
   2. Identify this topology. (1 point)
   3. Using the base $\mathcal{B}$, determine whether the following sets are open in $(\mathbb{R}, \mathcal{T})$:
      (a) $(\sqrt{2}, \pi)$, (0.5 point)
      (b) $(0, 1) \cup (1, 2)$, (0.5 point)
      (c) $\mathbb{Q}$. (0.5 point)

4. (4 points) Find the topology generated by the collection

$$\mathcal{S} = \{\{a\}, \{b\}, \{a, c, d\}\}$$

   on $X = \{a, b, c, d\}$.

5. (4 points) Let A and B be subsets of a topological space $(X, \mathcal{T})$. Show that:
   (a) $\text{Int}(A) \cup \text{Int}(B) \subseteq \text{Int}(A \cup B)$. (3 points)
   (b) Give an example to show that the reverse inclusion in (a) need not hold in general. (1 point)