

Q1: Let $\sigma = (143)(3547) \in S_8$.

(1) Write σ as a product of disjoint cycles, find its order, determine if σ is even or odd.

(2) Compute σ^{-1} and σ^{666} . What is the solution set to the equation $x^2 = \sigma$, for $x \in S_8$.

Q2: Show that no two of the following groups are isomorphic: S_4 , A_4 , D_6 , and $\mathbb{Z}_{12}$.

$$Q_1: (1) \sigma = (143)(3547) = (147)(35) \Rightarrow |\sigma| = \text{l.c.m.}(3, 2) = 6.$$

Since $\sigma = \text{even} \cdot \text{odd} \Rightarrow \sigma$ is odd.

$$(2) \sigma^{-1} = (174)(35) \quad \sigma^{666} = (\sigma^6)^{111} = e^{111} = e.$$

$x^2 = \sigma$ has no solution since σ is odd and x^2 is even for any $x \in S_8$.

$$Q_2: \text{For } S_4: |S_4| = 24 \neq 12 = |A_4| = |D_6| = |\mathbb{Z}_{12}|.$$

For $\mathbb{Z}_{12}$: The only abelian group.

For A_4 : A_4 has 3 elements of order 2 and D_6 has 7.