

Math 343	May 4, 2026	Midterm Exam 2	Time: 90 minutes
Name:	Student ID #		

Question #	1	2	3	4	5	Total
Score						

1. Justify answers and show all work for full credit.
2. Answer the questions in the space provided. If you run out of room continue on the back of the page.

Q1: Determine whether each statement is true or false and justify your answer. (**Choose 4 only**) [4 pts]

1. The set of odd permutations in S_n forms a subgroup.
2. The group $\mathbb{Z} \oplus \mathbb{Z}$ is cyclic.
3. The group $\mathbb{Z}_{16} \oplus \mathbb{Z}_{25} \oplus \mathbb{Z}_{999}$ is cyclic.
4. If $H, K \leq G$ with $|H| = 12$ and $|K| = 35$, then we must have $H \cap K = \{e\}$.
5. If $H, K \leq G$ with $|G| = 180$, $|H| = 10$, and $|K| = 20$, then we must have $H \cap K = \{e\}$.
6. The element $(2,4,6)$ in $\mathbb{Z}_6 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_{12}$ has the largest possible order.

[4pts]Q2: Let $\alpha = (13568)(2458) \in S_8$.

- (i) Write α as a product of disjoint cycles, find its order.
- (ii) Write α as a product of transpositions and determine if it is even or odd.
- (iii) Show that $\langle \alpha \rangle \cap A_8 = \langle \alpha^2 \rangle$. (Hint: Which powers of α are even permutations?)

- [4pts]Q3: (a) Define what it means for a map f to be an isomorphism from a group G to a group G' .
- (b) Determine with justification if any of the following groups are isomorphic: $\mathbb{R}^+$, $\mathbb{R}^*$, and $\mathbb{R}$.
- (c) Determine with justification if any of the following groups are isomorphic: D_6 , A_4 , and $\mathbb{Z}_2 \oplus \mathbb{Z}_6$.

[4 pts]**Q4:** How many elements of order 2 and 4 in each of the following groups: $\mathbb{Z}_2 \oplus \mathbb{Z}_2$, $\mathbb{Z}_2 \oplus \mathbb{Z}_4$, and $\mathbb{Z}_4 \oplus \mathbb{Z}_4$?

[4pts]**Q5:** Compute the order of each of the following groups and determine if it is cyclic or not:
 $\text{Aut}(\mathbb{Z})$, $\text{Aut}(\mathbb{Z}_9)$, $\text{Aut}(\mathbb{Z}_{20})$, and $\text{Inn}(\mathbb{Z}_{20})$.

