

Math 343	November 17, 2025	Midterm Exam 2	Time: 90 minutes
Name:	Student ID #		

Question #	1	2	3	4	5	Total
Score						

1. Justify answers and show all work for full credit.
2. Answer the questions in the space provided. If you run out of room continue on the back of the page.

Q1: Determine whether each statement is true or false and justify your answer. (**Choose 4 only**) [6 pts]

1. The set of odd permutations is closed under taking inverses.
2. If $m \neq n$, then $U(m) \not\cong U(n)$.
3. If N is a normal subgroup of G such that N and G/N are both cyclic then G must be cyclic.
4. If $m \mid |G|$ then G must have a subgroup of order m .
5. If G and G' are groups such that $G \oplus G'$ is abelian, then G and G' are both abelian.
6. $|(2,4,6)| = 30$ in $\mathbb{Z}_6 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_{12}$.

[4pts]Q2: Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 5 & 3 & 4 & 2 & 1 \end{pmatrix} \in S_7$.

- (i) Write the cycle decomposition of α , find its order and determine if its odd or even.
- (ii) write α^2 as a product of disjoint cycles.
- (iii) What the maximum possible order for an element in S_7 .

[3pts]**Q3:** (a) Let G be a group. Define what it means for (i) a map f to be an automorphism of G . (ii) a subgroup H to be normal.

(b) Compute the group $\text{Aut}(\mathbb{Z}_{100})$, is it cyclic? Explain why all its subgroups must be normal.

[4 pts] **Q4:** Let G be a finite group and $A, B \triangleleft G$.

(i) Prove that $A \cap B \triangleleft G$ and $[G : A \cap B] = [G : A][A : A \cap B]$.

(ii) Show that the map $\varphi: G/A \cap B \rightarrow G/A \oplus G/B$ given by $\varphi(g(A \cap B)) = (gA, gB)$ is a well-defined operation preserving one to one map.

(iii) Show that if $[G : A]$ and $[G : B]$ are relatively prime then φ is an isomorphism.

[3pts]**Q5:** Show that none of the following groups are isomorphic:

$$\mathbb{Z}_{24}, \mathbb{Z}_6 \oplus \mathbb{Z}_4, S_4, D_{12}, S_3 \oplus \mathbb{Z}_4, D_4 \oplus \mathbb{Z}_3$$