Math 343 – 14462, April 21, 2025	Midterm Exam 2	Time: 90 minutes
Name:	Student ID #	

Question	1	2	3	4	5	Total
Grade						

1. Justify answers and show all work for full credit.
2. Answer the questions in the space provided. If you run out of room continue on the back of the page.

Q1: Determine whether each statement is true or false and justify your answer. (**Choose 4 only**) [8pts]

1. Every subgroup of S_4 is normal.
2. There exists a finite group G of order 42 with subgroups H and K such that $|H| = 12$, $|K| = 14$ and $|H \cap K| = 2$.
3. There is a nonabelian group of order 11,
4. If every proper subgroup of G is abelian then G must be abelian.
5. The external direct product of cyclic groups must be cyclic.
6. Let G be a group of order pq, where p and q are prime numbers, then every subgroup of G is cyclic.

Q2: **(i)** Let $G = \langle a \rangle$, where $|a| = 24$. list all generators for the subgroup of order 8. [3pts]

(ii) Show that a group G that has more than $p - 1$ elements of order p cannot be cyclic.

[3pts] **Q3: (i)** Find all elements in $Aut(\mathbb{Z}_{24})$.

(ii) Find a cyclic subgroup of $\mathbb{Z}_{40} \oplus \mathbb{Z}_{30}$ of order 12 and a non-cyclic subgroup of order 12.

[3pts] **Q4:** Let ϕ be a group isomorphism from a group G, to a group $\bar{G}$, and let K be a normal subgroup of $\bar{G}$. Show that:

(i) $\phi^{-1}(K)$ is a normal subgroup of G.

(ii) $Ker(\phi)$ is a normal subgroup of G.

[3pts] **Q5**: Let G be a group and let G' be the subgroup of G generated by the set $S = \{x^{-1}y^{-1}xy | x, y \in G\}$

(i) Prove that G' is normal in G.

(ii) Prove that G/G' is Abelian.