

Math 343	March 2, 2026	Midterm Exam 1	Time: 90 minutes
Name:	Student ID #		

Question	1	2	3	4	Total
Grade					

1. Justify answers and show all work for full credit.
2. Answer the questions in the space provided. If you run out of room continue on the back of the page.

Q1: Determine whether each statement is true or false and justify your answer. (**Choose 4 only**) [5 pts]

1. If G is a group such that $(xy)^{-1} = x^{-1}y^{-1}$ for all $x, y \in G$, then G must be abelian.
2. There is an abelian group G with $|G| = n$ for any positive integer n .
3. Let G be an abelian group. If $a, b \in G$ both have finite order, then ab must have a finite order.
4. If a group G has an element of order 15, then G must have at least 8 elements of order 15.
5. If G is a group and $H, K \leq G$, then $H \cup K \leq G$.

[5pts]Q2: Let G be a group and $a \in G$.

- (a)
 - i) Define the centralizer $C(a)$.
 - ii) Show that $C(a) \leq G$.
 - iii) Show that $C(a^{-1}) = C(a)$.
- (b)
 - i) Define the center $Z(G)$.
 - ii) Show that $Z(G) \leq G$.
 - iii) Show that $Z(G) \subseteq C(a)$ for any $a \in G$.
- (c) Give an example of a group G and $a \in G$ such that $\{e\} \subsetneq Z(G) \subsetneq C(a) \subsetneq G$ is a sequence of **proper** subgroups.

[5pts]**Q3:** Determine if the following groups are cyclic and if so find all its generators:

(1) $\mathbb{Z}$

(2) $\mathbb{Q}^*$

(3) $\mathbb{Z}_{18}$

(4) $U(35)$

(5) D_4

[5pts]**Q4:** In $\mathbb{Z}_{36}$, find: (1) All elements of order 6. (2) All generators of $\langle 8 \rangle$. (3) A generator for $\langle 21 \rangle \cap \langle 10 \rangle$.

