

Math 343	1 October 2025	Midterm Exam 1	Time: 90 minutes
Name:	Student ID #		

Question	1	2	3	4	5	Total
Grade						

1. Justify answers and show all work for full credit.
2. Answer the questions in the space provided. If you run out of room continue on the back of the page.

Q1: Determine whether each statement is true or false and justify your answer. (**Choose 4 only**) [8 pts]

1. In any group G , the set of elements of order at most 2 forms a subgroup.
2. Every group of order 4 is cyclic.
3. Let G be a cyclic group of order n , and $k|n$ a positive integer. Then there exists a subgroup of order k .
4. A group that has no proper non-trivial subgroup is abelian.
5. Let G be a group, $a, b \in G$. Then $(ab)^2 = a^2b^2$ if and only if $ab = ba$.
6. There exists a cyclic group with three different elements of order 3.
7. There exists a group $G = \langle a, b \rangle$ which is not abelian.

[3pts]**Q2:** Let G be a group. Prove that

- (i) The identity e is unique.
- (ii) If $ab = e$ for elements a, b in G , then $a^{-1} = b$.
- (iii) If $G = \langle a \rangle$, then G is abelian.

[3pts]**Q3:** Let G be an abelian group and let $H = \{g^3 : g \in G\}$.

(i) Show that $H \leq G$.

(ii) Find the subgroup H for $G = \mathbb{Z}_6$, $G = U(8)$, and $G = \mathbb{Z}_{1000}$.

[3 pts] **Q4:** Let G be a group, $g \in G$, and $H \leq G$. Define $K = \{ghg^{-1} : h \in H\}$.

(i) Show that $K \leq G$.

(ii) Show that $|x| = |gxg^{-1}|$ for all $x, g \in G$. Deduce that H is cyclic *iff* K is cyclic.

[3pts] **Q5:** In $\mathbb{Z}_{24}$

- (i) List all elements of order 6.
- (ii) List all generators of $\langle 8 \rangle$.
- (iii) Find a generator for $\langle 21 \rangle \cap \langle 10 \rangle$.