Math 343 – 14462, February 17, 2025	Midterm Exam 1	Time: 90 minutes
Name:	Student ID #	

1. Justify answers and show all work for full credit.
2. Answer the questions in the space provided. If you run out of room continue on the back of the page.

Q1: Determine whether each statement is true or false and justify your answer. [5pts]

1. The set of all irrational numbers together with 0 under addition forms a group.
2. There exists $\sigma \in S_n$ such that $\sigma^2 = (12)$.
3. Let G be an abelian group and $x, y \in G$ have finite order, then xy has finite order.
4. A group with only a finite number of subgroups is finite.
5. There is an element of order 4 in $\mathbb{R}^*$ the group of nonzero real numbers under multiplication.

[6pts]Q2: a) Show that if $\alpha, \beta \in S_n$ commute and $i \in \{1,2,\dots,n\}$ is fixed by α , i.e. $\alpha(i) = i$ then $\beta(i)$ is also fixed by α.

b) Let $\sigma = (1643)(3547) \in S_8$.

(1) Write σ as a product of disjoint cycles and find its order.

(2) Write σ as a product of transpositions and determine if σ is even or odd.

(3) Compute σ^{-1} and σ^{22}.

[3pts]Q3: Prove that $(\mathbb{Z},*)$ is a group where $a * b = a + b - 1$.

[3pts]Q4: a) Let G be a group. (i) Define the center $Z(G)$ and show that it is a subgroup. (ii) Give an example of a group having a proper nontrivial center.

b) Let G be a group and H a nonempty subset that contains $x^{-1}y$ whenever $x, y \in H$. Show H is a subgroup.

[3pts]Q5: Let G be a group.

a) Show that $|ab| = |ba|$ for any $a, b \in G$.

b) Show that if $xy = x^{-1}y^{-1}$ for all $x, y \in G$ then G must be abelian.