

Math 343 – 472, June 7, 2026	Final Exam	Time: 3 hours
Name:	Student ID #	

1. Justify answers and show all work for full credit.
2. Answer the questions in the space provided. If you run out of room continue on the back of the page.

Question	1	2	3	4	5	Total
Grade						40

[9pts]Q1: Determine whether each statement is true or false and justify your answer. (Choose 6 only)

1. Every group of order pq (p, q are twin primes i.e. $q = p + 2$) is abelian.
2. It is possible for a quotient group G/N to be abelian even when G is nonabelian.
3. There exist at least two nonabelian groups of order 50 which are not isomorphic.
4. If $G/Z(G)$ is cyclic, then G is abelian.
5. Every subgroup of index 2 in a finite group is normal.
6. $\text{Inn}(D_4) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$.
7. The centralizer $C(a)$ of any element a in a group G is normal in G .
8. A_4 has a subgroup of order 6.
9. A Sylow p -subgroup is normal *iff* it is unique.

[7pts]Q2:

- a. Define what it means for two groups G and G' to be isomorphic.
- b. Let $\varphi: G \rightarrow G'$ be a group homomorphism. Show $\text{Im}(\varphi) \leq G'$ but not necessarily normal.
- c. Let $\varphi: G \rightarrow G'$ be a surjective group homomorphism. Show that if G is abelian, then G' is abelian.

[8pts]Q3:

- a. Let $\alpha \in S_9$ be an element of order 12. List all possible cycle decompositions of α and give an example for each one.
- b. Let G be a group and let $H, K \triangleleft G$. Prove that $HK \triangleleft G$.
- c. Show that every subgroup of a cyclic group is cyclic.

[9pts]Q4:

- a. Let $G = \mathbb{Z}_{12} \oplus \mathbb{Z}_{18}$.
 - i. Compute $|(9,8)|$.
 - ii. How many elements of order 6 does G have?
- b. Let G be an abelian group of order 360:
 - i. Classify (up to isomorphism) all abelian groups of order 360.
 - ii. Determine which ones have an element of order 12.
- c. Prove that a group of order $2p$, where p is an odd prime is isomorphic to $\mathbb{Z}_{2p}$ or D_p .

[7pts]Q5

- a. Define a Sylow p -subgroup of a finite group G .
- b. Let $|G| = 200$. Prove that G is not simple.
- c. Define what it means for two elements to be conjugates.
- d. Find the conjugacy class of the permutation $\alpha = (13)$ in S_3 .

