

Math 343 – 471, December 21, 2025	Final Exam	Time: 3 hours
Name:	Student ID #	

1. Justify answers and show all work for full credit.
2. Answer the questions in the space provided. If you run out of room continue on the back of the page.

Question	1	2	3	4	5	Total
Grade	9	8	8	8	7	40

[9pts]Q1: Determine whether each statement is true or false and justify your answer.(Choose 6 only)

1. The maximum order of an element in S_7 is 7.
2. If $\alpha, \beta \in S_n$ are conjugates, then they are both even or both odd.
3. $\text{Aut}(\mathbb{Z}_9) \approx \mathbb{Z}_6$.
4. If n is an odd integer, then $\mathbb{Z}_2 \oplus \mathbb{Z}_n$ is cyclic.
5. A group G is abelian if and only iff $|\text{Inn}(G)| = 1$.
6. A group cannot be isomorphic to one of its proper subgroups.
7. Let A and B be groups. If $A \oplus B$ is cyclic then both A and B must be cyclic.
8. A_4 is isomorphic to a subgroup of D_8 .

- [8pts]Q2: (a) Let G and G' be groups. Define what it means for a map $\varphi: G \rightarrow G'$ to be a homomorphism.
- (b) Show that if $H' \leq G'$, then $\varphi^{-1}(H') \leq G$.
- (c) Show that if $H' \triangleleft G'$, then $\varphi^{-1}(H') \triangleleft G$.
- (d) State the First Isomorphism Theorem, then use it to show that there is no surjective homomorphism from A_4 onto D_4 .

- [8pts] **Q3:** (a) Let $\alpha \in S_{10}$ be an element of order 6. What are the possible cycle decompositions of α ? Give an example for each possibility and determine if it is even or odd.
- (b) Let G be a group and $A, B \triangleleft G$. Show that $A \cap B \triangleleft G$. Deduce that $A/(A \cap B)$ is a subgroup of $G/(A \cap B)$. Is it also normal? Justify.

[8pts] **Q4:**(a) Let $G = \mathbb{Z}_8 \oplus \mathbb{Z}_9$. Compute $|(6,6)|$. How many elements of order 6 does G have?

(b) Classify all abelian groups of order 225. Show that every abelian group of order 225 must have an element of order 15.

- [7pts] **Q5:** (a) Define what it means for a subgroup of a finite group to be a Sylow p -subgroup.
 (b) Classify all Sylow p -subgroups of S_4 and decide whether each normal or not.
 (c) Prove that a group of order pq is cyclic, where p and q are odd primes.

