Q1 Consider the function f defined by

$$f(x) = \begin{cases} 1, & -1 \leq x < 0 \\ \frac{1}{2}, & x = 0 \\ x, & 0 < x \leq 1 \end{cases},$$

and

$$f(x+2) = f(x), \ x \notin [-1, 1].$$

(a) Sketch the function f on the interval $[-3, 3]$. What is the period of f?

(b) Find the Fourier series representation for f.

(c) Find the sum of the Fourier series at $x = -\frac{1}{4}$.

(d) Show that

$$\frac{\pi^2}{8} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}.$$

Q2 Expand the function

$$f(x) = |x|, \quad x \in [-1, 1]$$

in Legender Polynomials.

Q3 Solve the heat equation

$$u_t = u_{xx}, \qquad 0 < x < \pi, \ t > 0,$$

subject to the boundary and initial conditions

$$\begin{aligned} u(0,t) &= u(\pi,t) = 0, \qquad t > 0, \\ u(x,0) &= 10, \quad 0 < x < \pi. \end{aligned}$$

Good Luck
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