Math 280 **Second Exam** **Second Semester 1446**

(9) 1. (a) Use the definition to show that $\lim_{x\to 3} \frac{1}{x} = \frac{1}{3}$

(b) Show that $\lim_{x\to 0} \cos\frac{2}{x}$ does not exist.

(c) Let $f : (0,\infty) \to \mathbb{R}$. Prove that if $\lim_{x\to 0^+} f\left(\frac{1}{x}\right) = L$ then $\lim_{x\to\infty} f(x) = L$.

(9) 2. (a) Show that $\cos x = x$ has a solution in $(0, \pi/2)$.

(b) Let $f : (-1, 1) \to \mathbb{R}$ satisfy

$$|f(x)| \le |x| \quad \forall x \in (-1, 1)$$

Show that f is continuous at $x = 0$.

(c) Show that $f(x) = \frac{1}{x}$ is uniformly continuous on $[1,\infty)$, but not uniformly continuous on $(0, 1)$.

(8) 3. (a) Prove that f is differentiable at $x = 0$, and evaluate $f'(0)$.

$$f(x) = \begin{cases} x^3 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{Q}^c \end{cases}$$

(b) Use the mean value theorem to show that $|\cos x - \cos y| \le |x - y|$ for all $x, y \in \mathbb{R}$.

(c) Find $\lim_{x\to 0} \frac{\sin x - x}{x^3}$