College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Second Midterm Exam
Academic Year 1446 Hijri- SecondSemester

Exam Information معلومات الامتحان			
Course name	Introducion to real analysis		اسم المقرر
Course Code	280 Math		رمز المقرر
Exam Date	2025-04-16	1446-10-18	تاريخ الامتحان
Exam Time	08: 00 AM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.	G009		رقم قاعة الاختبار
Instructor Name	Prof. Haifa Bin Jebreen		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 6 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Question number	I	II	III	IV	Total
Mark					

Question I:

(a) Prove that the Harmonic series $\sum_n \frac{1}{n}$ is divergent using the definition of convergence of a series.

(b) Discuss the convergence of $\sum_{n\geq 3} \frac{ln\ n}{n}$. (You can use (a))

(c) Is $\sum_{n\geq 3} \frac{(-1)^n\ ln\, n}{n}$ conditionally convergent? Justify your answer.

Question II:

(a) Test the series $\sum_n \frac{3^n n!}{n^n}$ for convergence

(b) Prove that if $\sum_n x_n$ is convergent then $\lim_{n\to\infty} x_n = 0$.

(c) Is the converse of (b) true? Justify your answer.

Question III:

(a) Find the following limits if they exist.

(i) $\lim_{x \to \infty} sgn(x)$

(ii) $\lim_{x \to 0^-} \frac{1}{x}$

(b) Use the intermediate value theorem to find an open interval in which the equation

$$x^3 - 5x = 4$$

has a solution

Question IV:

(a) Let

$$f(x) = \begin{cases} x \sin \frac{1}{x^2}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

(i) Study the continuity of f on $\mathbb{R}$.

(ii) Are the discontinuities, if they exist, removable? Explain your answer.

(b) Prove that $f(x) = sgn\ x$ is uniformly continuous on $(0, \infty)$.