Math 280 **Second Exam** **First Semester 1446**

(9) 1. (a) Use the definition to show that $\lim_{x\to 1} x^2 = 1$

(b) Show that $\lim_{x\to 0} \cos\dfrac{1}{x}$ does not exist.

(c) If $\lim_{x\to c} f(x) = 0$ and g is a bounded function, prove that $\lim_{x\to c} f(x)g(x) = 0$.

(9) 2. (a) Let $f : (-1, 1) \to \mathbb{R}$ satisfy

$$|f(x)| \leq |x| \quad \forall x \in (-1, 1)$$

Show that f is continuous at $x = 0$.

(b) Show that $x^3 - 2x = 0$ has a solution in $[1, 2]$

(c) Let $f, g : [a, b] \to \mathbb{R}$ be continuous and $f(a) < g(a)$, $f(b) > g(b)$. Show that there is $c \in (a, b)$ such that $f(c) = g(c)$.

(7) 3. (a) Show that $f : \mathbb{R} \longrightarrow \mathbb{R}$

$$f(x) = \begin{cases} x \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is continuous at $x = 0$, but not differentiable at $x = 0$.

(b) Use the mean value theorem to show that

$$\sin x \leq x \qquad \forall x \geq 0$$

Good Luck