Question number	I	II	III	IV	Total
Mark					

Question I

(a) Prove that if (x_n) is a Cauchy sequence then it is bounded.

(b) Is the converse of (a) true? Justify your answer.

(c) Prove that if x is a cluster point of a subset A of R, then there is a sequence (a_n) in A such that $a_n \neq x$, and (a_n) converges to x.

Question II

(a) Prove that if $\sum_n x_n$ is convergent then $\lim x_n = 0$.

(b) Is the converse of (a) true? Justify your answer.

Question III

Test the following series for convergence:

(a) $\sum_n (-1)^n \frac{\sqrt{n}}{n^2}$

(b) $\sum_n \frac{1}{(n^3-1)}$

(c) $\sum_n \frac{2^n n!}{n^n}$

Question IV

Find the limit if it exists:

a) $\lim_{x \to 5} f(x)$, where $f(x) = \begin{cases} 1, & x \in Q \\ -1, & x \notin Q \end{cases}$

b) $\lim\limits_{x \to 0} \left(\cos \dfrac{1}{2x} \right)$

c) $\lim\limits_{x \to 2} (x \, sgn \, x)$