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- (2) 1. (a) Determine $\sup A$ and $\inf A$

$$A = \left\{ n \in \mathbb{N} : (-1)^n + \frac{(-1)^n}{n} \right\}$$

- (4) (b) Let A and B be bounded subsets of \mathbb{R} . Prove that

$$\sup(A + B) = \sup A + \sup B$$

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- (4) 2. (a) Prove by definition that

$$\lim \frac{3n^2 + 2}{2n^2 + 3} = \frac{3}{2}$$

- (4) (b) Show that the sequence (x_n) is bounded and monotone; therefore, convergent then find its limit where $x_1 = 1$,

$$x_{n+1} = \sqrt{7x_n}$$

- (4) (c) If $\lim \frac{x_n - 1}{x_n + 1} = 0$ prove that $\lim x_n = 1$

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- (4) 3. (a) Determine whether the series converges (if it does is it conditionally or absolutely) or diverges

1.

$$\sum_{n=1}^{\infty} \frac{4^n}{n^4}$$

2.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n + \sqrt{n}}$$

- (3) (b) Let the sequence (x_n) be positive and decreasing. If $\sum x_n$ converges, prove that $\lim nx_n = 0$
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