<u>Question I</u> (6 marks)

(a) Determine Sup(A), Inf(A), Max(A) and Min(A) if exist, where

$$A = \left\{\frac{1}{n} - 1 : n \in N\right\}.$$

(b) Let A and B be two bounded below subsets of R such that $A \subseteq B$.

(i) Prove that Inf A $\geq$ Inf B

(ii) Can Inf A = Inf B? Justify your answer.

Question II (8 marks)

(a) Find and prove the following limits if exist, if not, justify your answer

(i) $\lim_{n\to\infty} \frac{n^2}{2n^2+1}$.

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(ii) $\lim_{n\to\infty} \left(\frac{3}{5}\right)^n$.

(b) Find $\widehat{Q^c}$, the cluster point set of the irrational numbers. Justify your answer.

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(c) Prove that (x_n) where $x_n = \frac{2n}{3n+1}$ is a Cauchy sequence.

<u>Question III</u> (5 marks)

(a) Let $x_1 = 1, x_{n+1} = \sqrt{2x}\ for\ all\ n \in N.$

(i) Prove (x_n) is monotone.

(ii) Prove (x_n) is bounded.

(iii) Find the limit of (x_n).

<u>Question IV</u> (6 marks)

Prove or disprove the following:

(a) Every Cauchy sequence in Q is convergent in Q.
(b) Every sequence that has a convergent subsequence is bounded.
(c) There are sequences (x_n) and (y_n) such that $x_n < y_n$, but $\lim x_n = \lim y_n$.
(d) $If \quad lim\, x_n = \mathrm{c}$, then $lim|x_n| = |c|$.