Math 280 **First Exam** **First Semester 1446**

(2) 1. (a) Determine $\sup A$ and $\inf A$

$$A = \left\{ n \in \mathbb{N} : 2(-1)^{n+1} + \frac{(-1)^n}{n} \right\}$$

(4) (b) If A and B are nonempty bounded sets and $A \subset B$ show that $\sup A \leq \sup B$

(4) 2. (a) Prove by definition that

$$\lim \frac{n^2+2}{3n^2+n} = \frac{1}{3}$$

(4) (b) Show that the sequence (x_n) is bounded and monotone; therefore, convergent then find its limit where $x_1 = 1$,

$$x_{n+1} = \sqrt{5x_n}$$

(4) (c) Show that (x_n) is a Cauchy sequence if it satisfies the condition

$$|x_{n+1} - x_n| < \frac{1}{3^n}$$

(4) 3. (a) Determine whether the series converges or diverges

1.
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + 2^n}$$

2.
$$\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$$

(3) (b) If $x_n > 0$ and the series $\sum_{n=1}^{\infty} x_n$ diverges, show that

$$\sum_{n=1}^{\infty} \frac{x_n}{1+x_n}$$

diverges.

[Hint: Consider the two cases (if $x_n \not\to 0$ and if $x_n \to 0$. In the second case show that there is $N \in \mathbb{N}$ such that

$$\frac{x_n}{2} \leq \frac{x_n}{1+x_n} \leq x_n \qquad \forall n \geq N$$

then conclude that the series diverges.)].