Question I

(a) (i) Determine Sup(A) and Inf(A), where $A = \left\{1 - \frac{1}{n^2} : n \in N\right\}$, Justify your answer.

(ii) Is 3 an upper bound of A? Justify your answer.

(b) Prove that for every positive real number x, there is a natural number n such that $0 < \frac{1}{n} < x$.

Question II

Find and prove the following limits.

(i) $\lim_{n\to\infty} \frac{(-1)^n}{n}$.

(ii) $\lim_{n\to\infty}(n^2+1)$.

(iii) $\lim_{n\to\infty}\frac{2n}{n+2}$

Question III

(a) Prove that every convergent sequence has a unique limit.

(b) Prove that $lim|x_n| = 0 \; if \; and \; only \; if \; lim\, x_n$=0.

Question IV:

Let $x_1 = 1, x_{n+1} = \sqrt{x_n + 3}. \; for \; all \; n \in N.$

(a) Prove (x_n) is monotone.
(b) Prove (x_n) is bounded.
(c) Find the limit of (x_n).