Math 280 **Final Exam** **Second Semester 1446**

1. **Question 1** [2+4+4+3=13]
   (a) Determine $\sup A$ and $\inf A$ where $A = \left\{2 + \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$
   (b) Prove by definition that $\lim \dfrac{3n+1}{2n+2} = \dfrac{3}{2}$
   (c) Prove that (x_n) is a Cauchy sequence if
   $$|x_{n+1} - x_n| < \frac{1}{2^n}$$
   (d) Determine weather the series is convergent or not
   $$\sum \frac{n^3}{3^n}$$

2. **Question 2** [4+3+3+4=14]
   (a) Find the following limits it they exist.
      1. $\lim\limits_{x\to 0} \dfrac{x^2(1+\sin x)}{(x+\sin x)^2}$
      2. $\lim\limits_{x\to 0^+} \left(1 + \dfrac{2}{x}\right)^x$
   (b) Let $f, g : [a,b] \to \mathbb{R}$ be continuous and $f(a) < g(a)$, $f(b) > g(b)$. Show that there is $c \in (a,b)$ such that $f(c) = g(c)$.
   (c) Show that the function $f(x) = x^2$ is not uniformly continuous on $\mathbb{R}$.
   (d) Use the definition to show that $f(x) = \sqrt{x^2+1}$ is differentiable on $\mathbb{R}$, then prove that there is $c \in (0,1)$ such that
   $$\sqrt{2} - 1 = \frac{c}{\sqrt{c^2+1}}$$

3. **Question 3** [5+4+4=13]

4. (a) Determine wether f is Riemann integrable over $[0,1]$ and evaluate the integral (if it exists) using Riemann sum.
      1. $f(x) = 2x - 1$
      2. $f(x) = \begin{cases} x & x \in \mathbb{Q} \\ 0 & x \in \mathbb{Q}^c \end{cases}$
   (b) Determine the pointwise limit of $f_n(x) = \dfrac{x}{nx+1}$ on $[0,1]$, then decide whether the convergence is uniform.
   (c) Discuss the uniform convergence of the series $\sum\limits_{n=1}^{\infty} \cos\left(\dfrac{x}{n^2}\right), \quad x \in [-5, 2]$

Good Luck