College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Final Exam
Academic Year 1446 Hijri- SecondSemester

Exam Information معلومات الامتحان			
Course name	Introducion to real analysis		اسم المقرر
Course Code	244 Math		رمز المقرر
Exam Date	2025-05-15	1446-11-17	تاريخ الامتحان
Exam Time	08: 00 AM		وقت الامتحان
Exam Duration	3 hours	ثلاث ساعات	مدة الامتحان
Classroom No.	G050		رقم قاعة الاختبار
Instructor Name	Prof. Haifa Bin Jebreen		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 1 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
-

:تعليمات عامة

- عدد صفحات الامتحان 1 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
-

هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	1.1	1(a)		
2	1.2	2,3		
3	2.1	2,3		
4	2.2	1,2,3		
5				
6				
7				
8				

EXAM COVER PAGE

Math 280 **Final Exam** **Second Semester 1446**

1. **Question 1** [2+4+4+3=13]
   (a) Determine $\sup A$ and $\inf A$ where $A = \left\{2 + \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$
   (b) Prove by definition that $\lim \dfrac{3n+1}{2n+2} = \dfrac{3}{2}$
   (c) Prove that (x_n) is a Cauchy sequence if
   $$|x_{n+1} - x_n| < \frac{1}{2^n}$$
   (d) Determine weather the series is convergent or not
   $$\sum \frac{n^3}{3^n}$$

2. **Question 2** [4+3+3+4=14]
   (a) Find the following limits it they exist.
      1. $\lim\limits_{x\to 0} \dfrac{x^2(1+\sin x)}{(x+\sin x)^2}$
      2. $\lim\limits_{x\to 0^+} \left(1 + \dfrac{2}{x}\right)^x$

   (b) Let $f, g : [a, b] \to \mathbb{R}$ be continuous and $f(a) < g(a)$, $f(b) > g(b)$. Show that there is $c \in (a, b)$ such that $f(c) = g(c)$.
   (c) Show that the function $f(x) = x^2$ is not uniformly continuous on $\mathbb{R}$.
   (d) Use the definition to show that $f(x) = \sqrt{x^2+1}$ is differentiable on $\mathbb{R}$, then prove that there is $c \in (0, 1)$ such that
   $$\sqrt{2} - 1 = \frac{c}{\sqrt{c^2+1}}$$

3. **Question 3** [5+4+4=13]

4. (a) Determine wether f is Riemann integrable over $[0, 1]$ and evaluate the integral (if it exists) using Riemann sum.
      1. $f(x) = 2x - 1$
      2. $f(x) = \begin{cases} x & x \in \mathbb{Q} \\ 0 & x \in \mathbb{Q}^c \end{cases}$

   (b) Determine the pointwise limit of $f_n(x) = \dfrac{x}{nx+1}$ on $[0, 1]$, then decide whether the convergence is uniform.
   (c) Discuss the uniform convergence of the series $\sum\limits_{n=1}^{\infty} \cos\left(\dfrac{x}{n^2}\right), \quad x \in [-5, 2]$

Good Luck