Math 280 **Final Exam** **First Semester 1446**

1. **Question 1** [2+4+4+3=13]
   (a) Determine $\sup A$ and $\inf A$ where $A = \left\{2 - \frac{1}{n} : n \in \mathbb{N}\right\}$
   (b) Prove by definition that $\lim \dfrac{n+2}{3n+1} = \dfrac{1}{3}$
   (c) Show that the sequence (x_n) is bounded and monotone; therefore, convergent then find its limit where $x_1 = 1$, and $x_{n+1} = \sqrt{3 + 2x_n}$
   (d) Determine weather the series is convergent or not
   $$\sum \frac{n^3}{n!}$$

2. **Question 2** [6+4+4=14]
   (a) Find the following limits it they exist.
      1. $\lim\limits_{x \to 0} \dfrac{|\sin x|}{x}$
      2. $\lim\limits_{x \to 0^+} \dfrac{\log(1+x)}{\sin x}$
   (b) If $f : [0,1] \longrightarrow [0,1]$ is continuous, show that f has a fixed point $c \in [0,1]$, i.e., $f(c) = c$.
   (c) If the function f satisfies $|f(x)| \leq |x|^2$. Prove that f is differentiable at $x = 0$.

3. **Question 3** [5+4+4=13]

4. (a) Determine wether f is Riemann integrable over $[0,1]$ and evaluate the integral (if it exists) using Riemann sum.
      1. $f(x) = x - 4$
      2. $f(x) = \begin{cases} x & x \in \mathbb{Q} \\ 1 & x \in \mathbb{Q}^c \end{cases}$
   (b) Determine the pointwise limit of $f_n(x) = x^n(1-x)$ on $[0,1]$, then decide whether the convergence is uniform.
   (c) Discuss the uniform convergence of the series
   $$\sum \frac{\sin nx}{n^2}$$

Good Luck