College of Science. كلية العلوم
Department of Mathematics قسم الرياضيات

Final Exam
Academic Year 1445 Hijri- SecondSemester

Exam Information معلومات الامتحان			
Course name	**280**		اسم المقرر
Course Code	**Math**		رمز المقرر
Exam Date	**2024-05-21**	**1445-11-13**	تاريخ الامتحان
Exam Time	**01: 00 PM**		وقت الامتحان
Exam Duration	**3 hours**	ثلاث ساعات	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name	**Dr. Haifa Bin Jebreen**		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 7 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 7 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	1.1 Explain fundamental concepts of real Analysis.	Q1,Q3(1)(2)		
2	1.2 Describe some properties of functions.	Q2,Q3		
3	2.1 Models problems with functions	Q2,Q3,Q4		
4	2.2 Solve problems of convergence, limit, continuity and differentiability.	Q1,Q2,Q3,		
5				
6				
7				
8				

EXAM COVER PAGE

Question I. [2+3+3+3]

(1) Let $A = \left\{\frac{n}{n+1} : n \in \mathbb{N}\right\}$. Determine $\sup(A)$ and $\inf(A)$ if they exist. Justify your answer.

(2) Test the convergence of the following series:

$$i) \sum_{n=1} \frac{\sin n}{(1.001)^n},$$

$$ii) \sum_{n=1} \left(\frac{n+1}{2n+1}\right)^n,$$

$$iii) \sum_{n=1} \frac{n^3}{n!}.$$

Question II. [2+2+2+4]

(1) Evaluate the following limits:
   (a) $\lim_{x\to-\infty} \frac{x}{|x|}$.
   (b) $\lim_{x\to2^+} \frac{x}{x-2}$.
   (c) $\lim_{x\to0^+} x^{\sin x}$.

(2) Let

$$f(x) = \begin{cases} \frac{\sin x}{|x|} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Study the continuity of f on $\mathbb{R}$. Justify your answer.

Question III. [3+3+3+3]

(1) Let $f : I \to \mathbb{R}$ be differentiable at $c \in I$. Prove that f is continuous at c.

(2) If the function f has an extremum on the interval (a, b) at the point c and f is differentiable at c, then $f'(c) = 0$.

(3) Show that:

a) $f(x) = x^2$ is not uniformly continuous on $[1, \infty)$.

b) $f(x) = \sin x$ is uniformly continuous on $\mathbb{R}$.

Question IV. [3+2+2]

(1) Evaluate $\int_0^1 x^2 \, dx$ using Riemann sums.

(2) Prove that if f is Riemann integrable, then $|f|$ is Riemann integrable.

(3) Is the converse of (2) true? Justify your answer.