Find the orthogonal Trajectories of the family of curves

12. $y^2 = c(1 - x^2)$

****

21. Find the member of the orthogonal trajectories for $x^2 + 3y^2 = cy$ that passes through $(1, 2)$.

****

10) The population of a town grows at a rate proportional to the population at time t.The initial population of 500 increases by 15% in 10 years.What will be the population in 30 years?.

****

11) The population of a town grows at a rate proportional to the population at time. The initial population P_0 becomes double after 50 years.When the population becomes $4P_0$?.

****

13) Initially there were 10 mg. of radioactive material present. After 2 months the mass decreased by 5%. If the rate of decay is proportional to the amount present at any time , then determine the falf-life of this material.

****

16) If 0.5% of radium disappears in 12 years .Find what percentage will disappear in 1000 years?.What is the half-life of radium?.

****

18) A hot iron rod was left in a room where the temperature was 20^0C.After one minute the temperature of the rod was recorded 35^0C.,and after two minutes it was 27.5^0C. What was the initial
temperature of the rod?.

****

22) A body initially at 50^0C is put into a 375^0C oven. After 75 minutes it is found that the temperature of the body is 125^0C. How long the body will take to attain the temperature 150^0C?.

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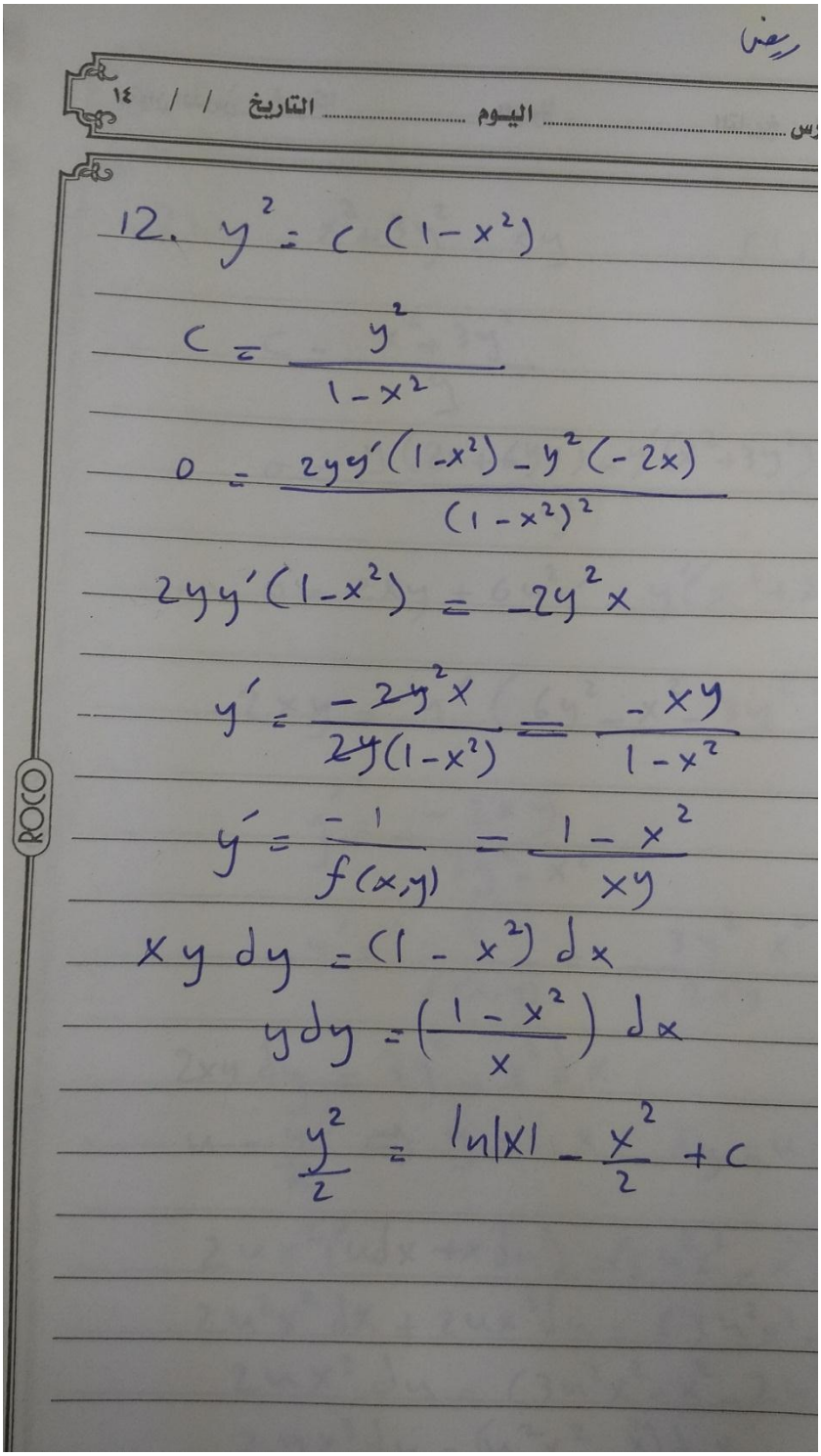
Find the orthogonal Trajectories of the family of curves

12. $y^2 = c(1 - x^2)$



12. $y^2 = c(1-x^2)$

$$c = \frac{y^2}{1-x^2}$$

$$0 = \frac{2yy'(1-x^2) - y^2(-2x)}{(1-x^2)^2}$$

$$2yy'(1-x^2) = -2y^2x$$

$$y' = \frac{-2y^2x}{2y(1-x^2)} = \frac{-xy}{1-x^2}$$

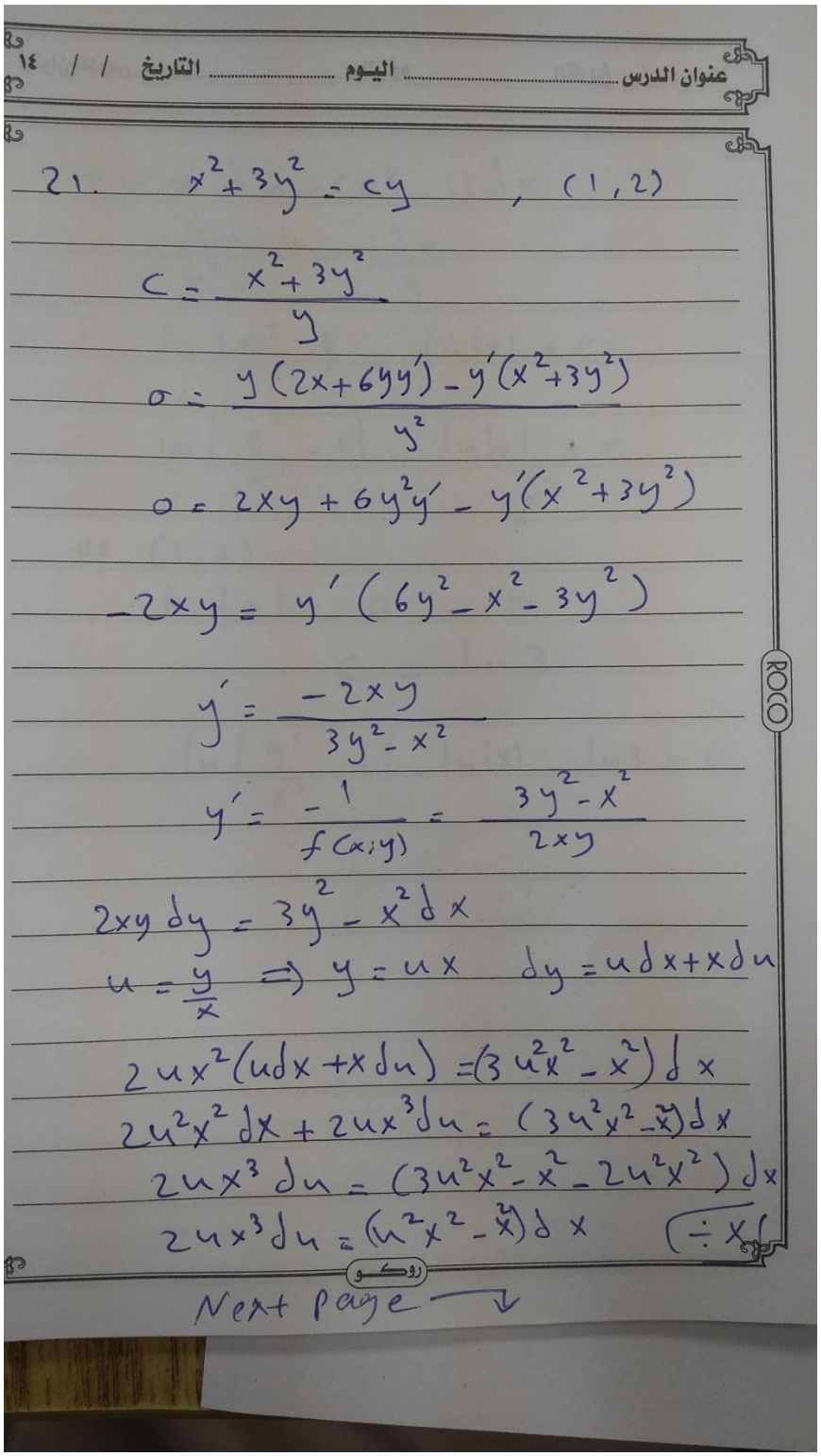
$$y' = \frac{-1}{f(x,y)} = \frac{1-x^2}{xy}$$

$$xy\,dy = (1-x^2)\,dx$$

$$y\,dy = \left(\frac{1-x^2}{x}\right)dx$$

$$\frac{y^2}{2} = \ln|x| - \frac{x^2}{2} + c$$

21. Find the member of the orthogonal trajectories for $x^2+3y^2 = cy$ that passes through $(1, 2)$.



$$2u\,x\,du = (u^2-1)\,dx$$

$$\frac{2u}{u^2-1}\,du = \frac{dx}{x}$$

$$\ln|u^2-1| = \ln|x| + C$$

$$\ln\left|\frac{y^2}{x^2}-1\right| - \ln|x| = C$$

at $(1,2)$

$$\ln|3| - 0 = C$$

$$C = \ln 3$$

$$\ln\left|\frac{y^2}{x^2}-1\right| - \ln|x| - \ln 3 = 0$$

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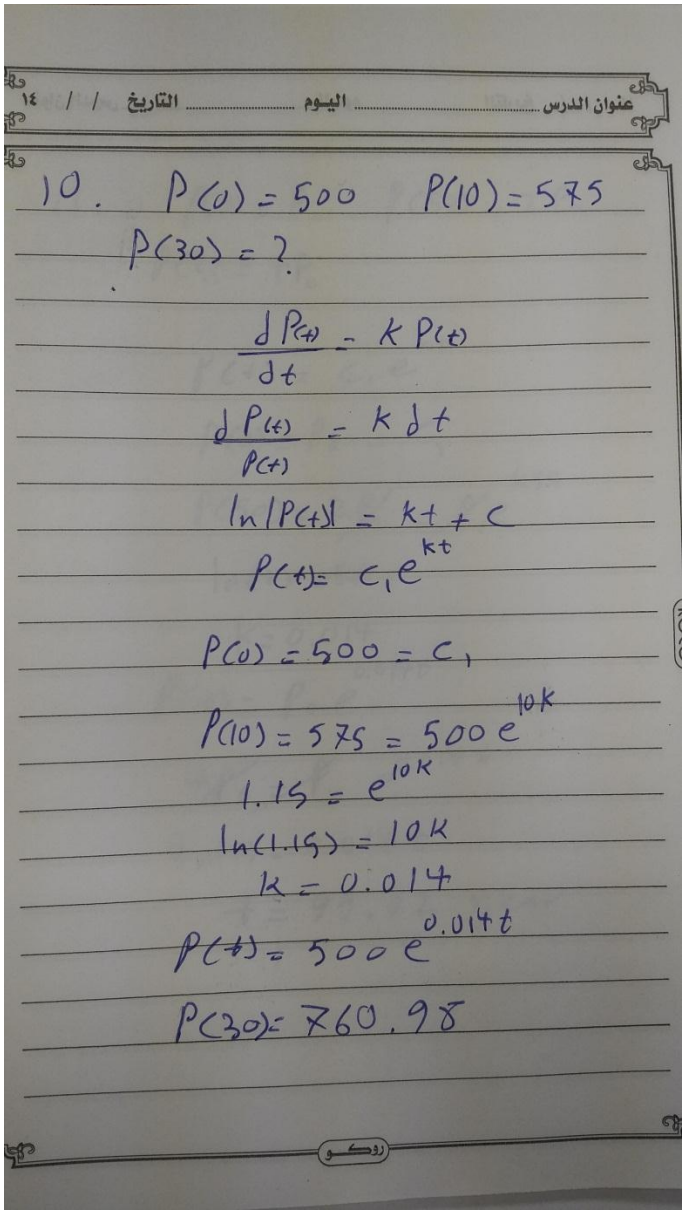
10) The population of a town grows at a rate proportional to the population at time t. The initial population of 500 increases by 15% in 10 years. What will be the population in 30 years?.



10. $P(0) = 500 \quad P(10) = 575$

$P(30) = ?$

$$\frac{dP(t)}{dt} = kP(t)$$

$$\frac{dP(t)}{P(t)} = k\,dt$$

$$\ln|P(t)| = kt + C$$

$$P(t) = c_1 e^{kt}$$

$$P(0) = 500 = c_1$$

$$P(10) = 575 = 500\,e^{10k}$$

$$1.15 = e^{10k}$$

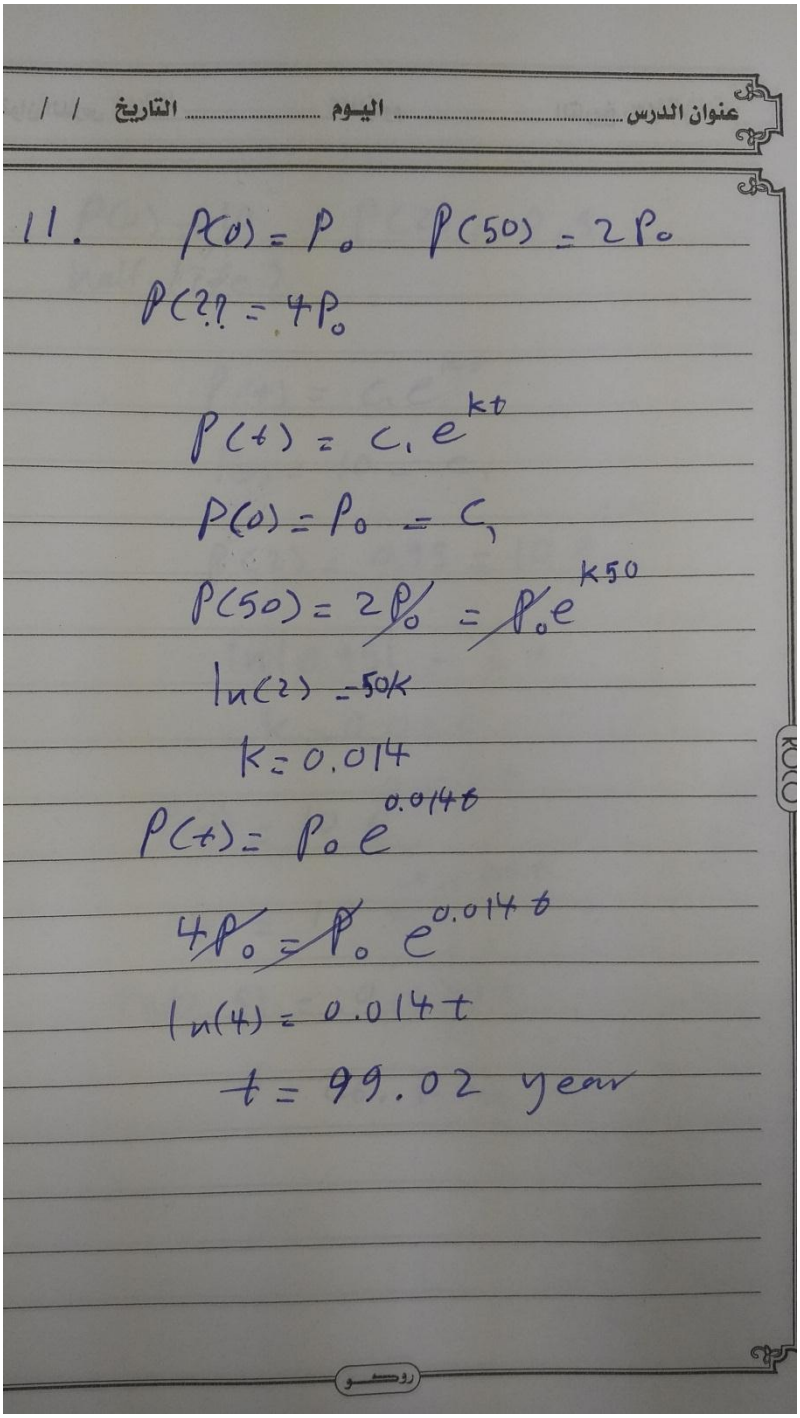
$$\ln(1.15) = 10k$$

$$k = 0.014$$

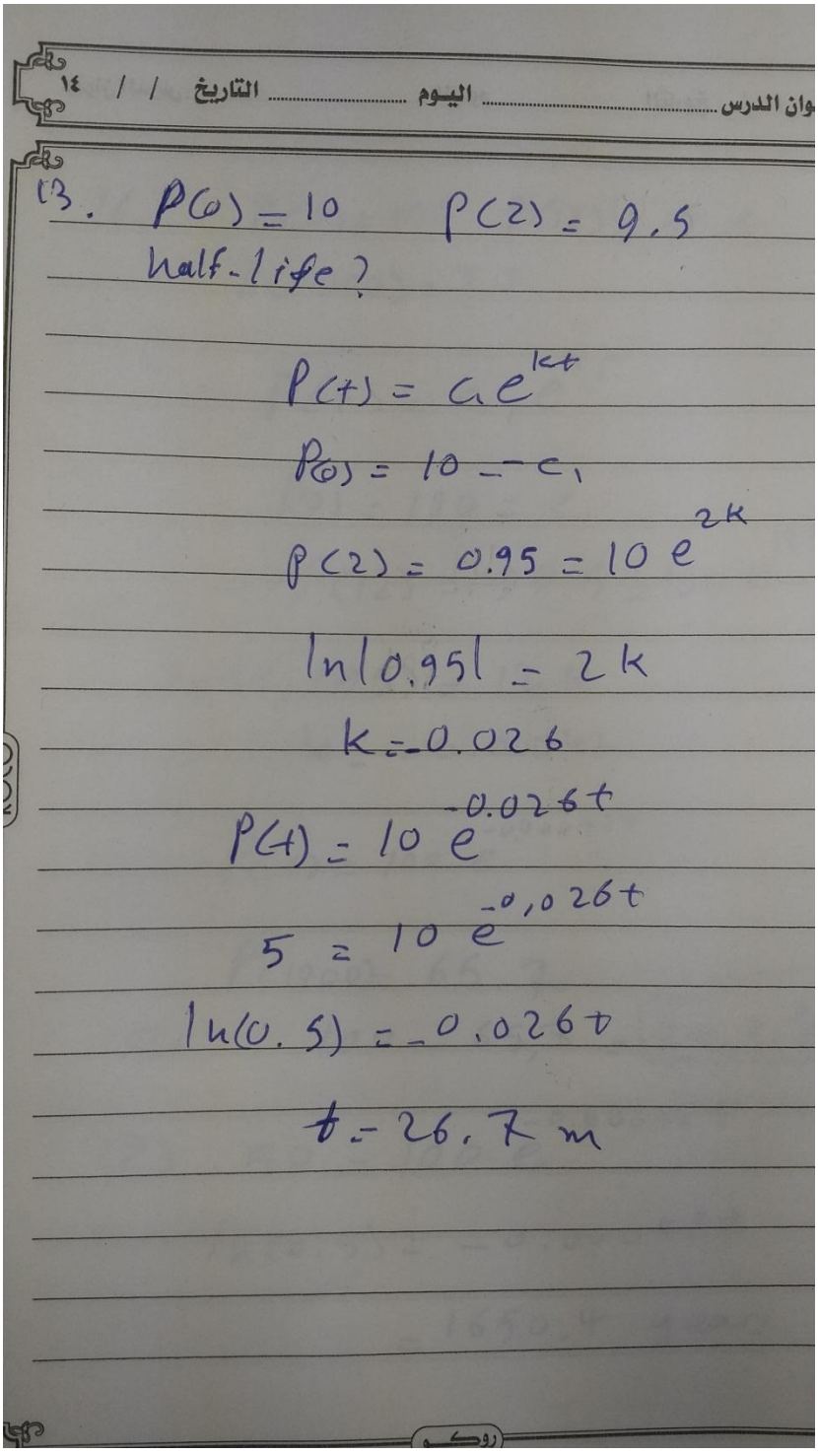
$$P(t) = 500\,e^{0.014t}$$

$$P(30) = 760.98$$

11) The population of a town grows at a rate proportional to the population at time. The initial population P_0 becomes double after 50 years.When the population becomes $4P_0$?.



13) Initially there were 10 mg. of radioactive material present. After 2 months the mass decreased by 5%. If the rate of decay is proportional to the amount present at any time , then determine the falf-life of this material.



13. $P(0) = 10 \quad P(2) = 9.5$

half-life ?

$$P(t) = ae^{kt}$$

$$P(0) = 10 = c_1$$

$$P(2) = 0.95 = 10e^{2k}$$

$$\ln|0.95| = 2k$$

$$k = -0.026$$

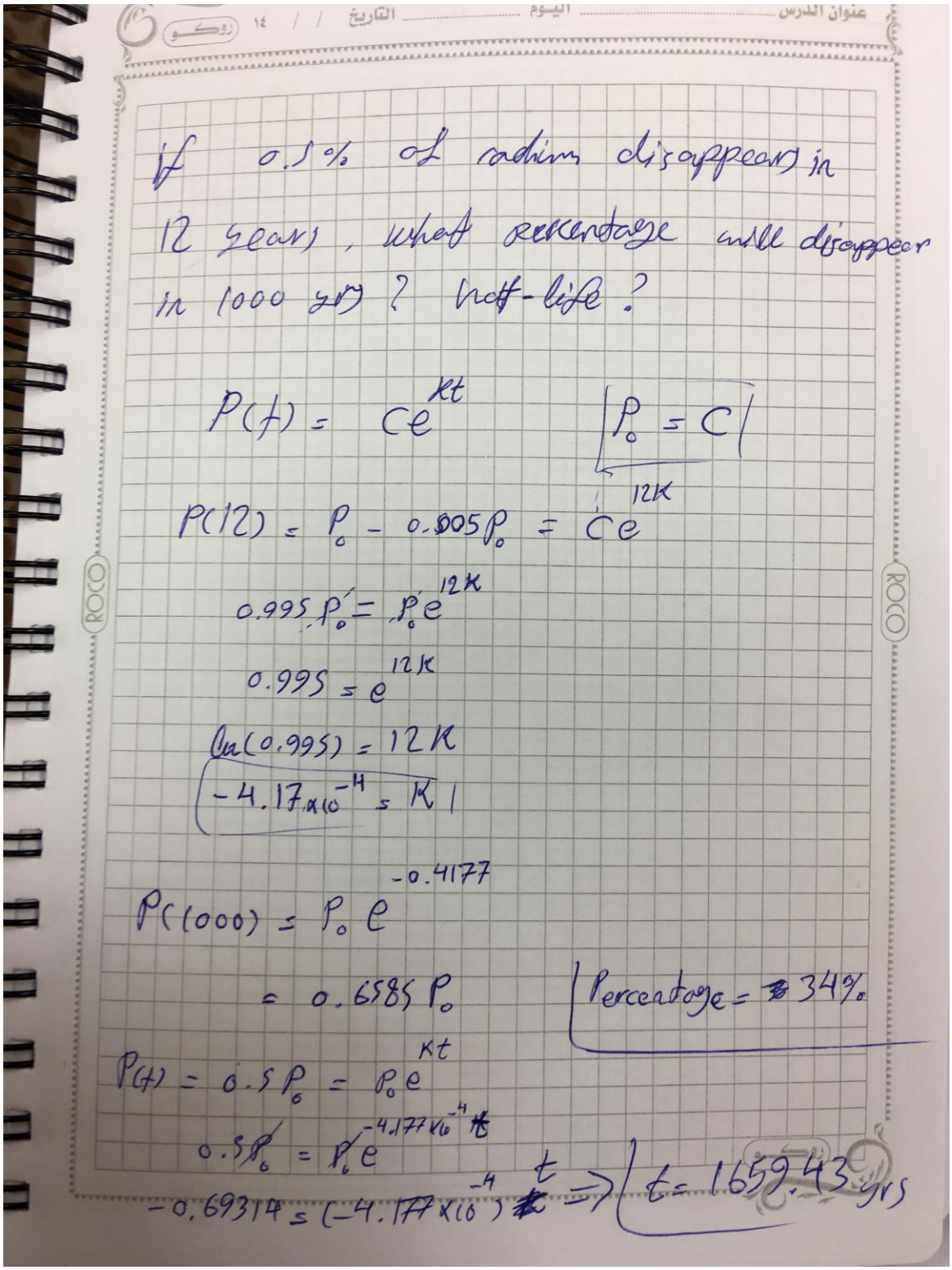
$$P(t) = 10e^{-0.026t}$$

$$5 = 10e^{-0,026t}$$

$$\ln(0.5) = -0.026t$$

$$t = 26.7 \text{ m}$$

16) If 0.5% of radium disappears in 12 years .Find what percentage will disappear in 1000 years?.What is the half-life of radium?.



If 0.5% of radium disappears in 12 years, what percentage will disappear in 1000 yrs? half-life?

$P(t) = Ce^{kt}$ $\quad P_0 = C$

$P(12) = P_0 - 0.005P_0 = Ce^{12k}$

$0.995P_0 = P_0e^{12k}$

$0.995 = e^{12k}$

$\ln(0.995) = 12k$

$-4.17 \times 10^{-4} = k$

$P(1000) = P_0 e^{-0.4177}$

$= 0.6585 P_0$ $\quad$ Percentage = 34%

$P(t) = 0.5P_0 = P_0e^{kt}$

$0.5P_0 = P_0e^{-4.177 \times 10^{-4} t}$

$-0.69314 = (-4.177 \times 10^{-4})t \Rightarrow t = 1659.43$ yrs

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16. $P(0) = 100 \quad P(12) = 99.5$

$P(1000) = ??$

$P(t) = C_1 e^{kt}$

$P(0) = 100 = C$

$P(12) = 99.5 = 100 e^{12k}$

$\ln(0.995) = 12k$

$k = -0.00042$

$P(t) = 100 e^{-0.00042t}$

$P(1000) = 65.7$

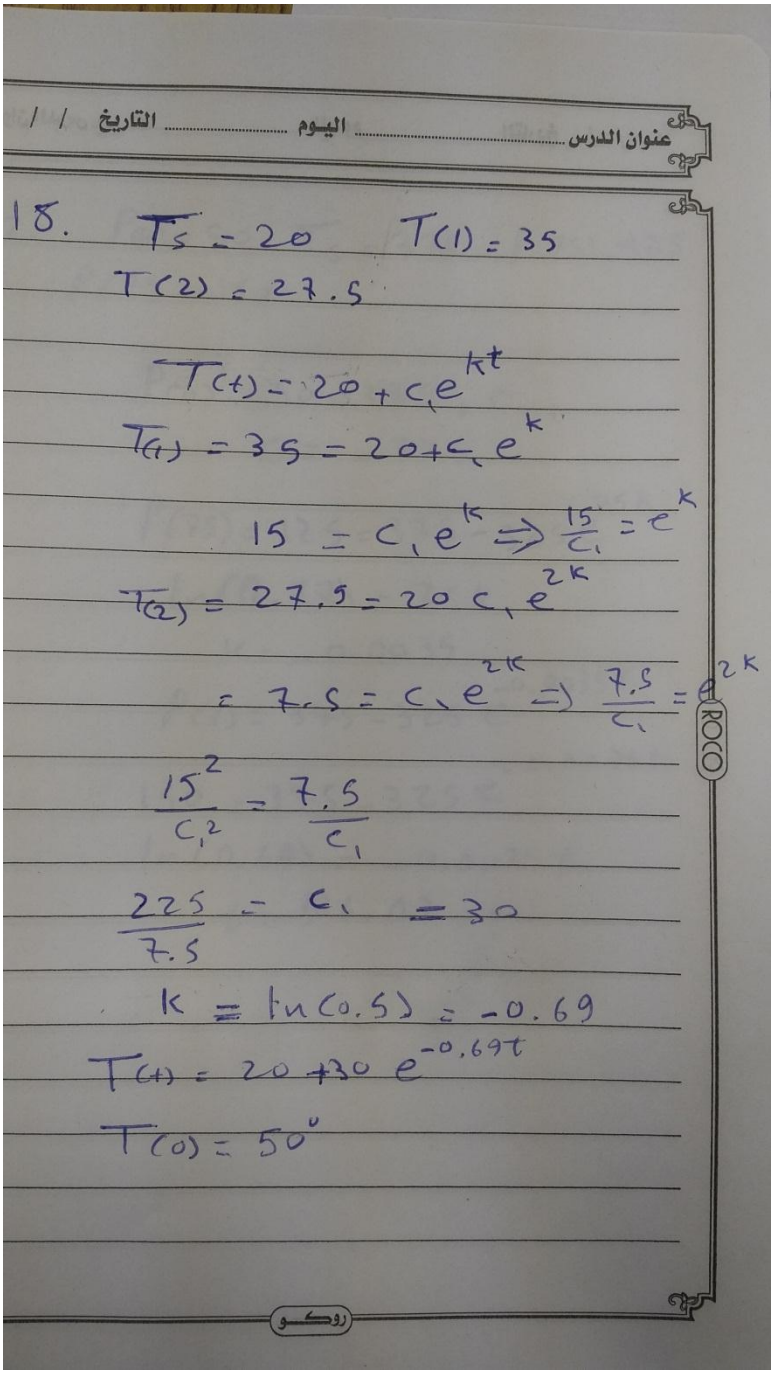
① $100 - 65.7 = \boxed{34.3\%}$

② $50 = 100 e^{-0.00042t}$

$\ln(0.5) = -0.00042t$

$= 1650.4$ years

18) A hot iron rod was left in a room where the temperature was 20^0C. After one minute the temperature of the rod was recorded 35^0C., and after two minutes it was 27.5^0C. What was the initial temperature of the rod?.



22) A body initially at 50^0C is put into a 375^0C oven. After 75 minutes it is found that the temperature of the body is 125^0C. How long the body will take to attain the temperature 150^0C?.