College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

**Final Exam**
**Academic Year 1447 Hijri- SecondSemester**

| Exam Information معلومات الامتحان | | | |
|---|---|---|---|
| **Course name** | **Vector Calculus حساب المتجهات** | | اسم المقرر |
| **Course Code** | **202 ريض** | | رمز المقرر |
| **Exam Date** | **2026-06-09** | **1447-12-23** | تاريخ الامتحان |
| **Exam Time** | **08: 00 AM** | | وقت الامتحان |
| **Exam Duration** | **3 hours** | ثلاث ساعات | مدة الامتحان |
| **Classroom No.** | **G03** | | رقم قاعة الاختبار |
| **Instructor Name** | هناء السديس | | اسم استاذ المقرر |

| Student Information معلومات الطالب | | |
|---|---|---|
| **Student's Name** | | اسم الطالب |
| **ID number** | | الرقم الجامعي |
| **Section No.** | | رقم الشعبة |
| **Serial Number** | | الرقم التسلسلي |

**General Instructions:**

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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**تعليمات عامة:**

- عدد صفحات الامتحان 6 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة
*This section is ONLY for instructor*

| # | Course Learning Outcomes (CLOs) | Related Question (s) | Points | Final Score |
|---|---|---|---|---|
| 1 | 1.3 | II(i-iii)+III(A) | /10 | |
| 2 | 2.1 | I(1,2) | /3 | |
| 3 | 2.2 | I(3-6)+II(iv-vi) | /11 | |
| 4 | 2.3 | III(B)+IV | /16 | |
| 5 | | | | /40 |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |

| Question | I(A) | I(B+C) | II(i-iii) | II(iv-vi) | III(A) | III(B) | IV | Total |
|---|---|---|---|---|---|---|---|---|
| Mark | | | | | | | | |

| Question | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Answer | | | | | | |

**Question I:** [1.5×6=9 marks]

Choose the correct answer, then fill in the table above.

**1**. The **symmetric equations of the line** that passes through the point $(3,-2,1)$ and perpendicular to the plane with equation $2x-y+4z-2=0$ are:

**(a)** $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{4}$.

**(b)** $\frac{x+3}{2}=\frac{y-2}{-1}=\frac{z+1}{4}$.

**(c)** $\frac{x-2}{3}=\frac{y-2}{-2}=\frac{z-4}{1}$.

**(d)** $\frac{x-3}{3}=\frac{y+2}{-2}=\frac{z-1}{1}$.

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**2.** The **equation of the plane** passing through the point $(1,5,0)$ and perpendicular to the line

$$L \equiv \begin{cases} x = 1-2t \\ y = 2-t, \\ z = 4+3t \end{cases} \qquad t \in \mathbb{R}, \quad \text{is:}$$

**(a)** $-2x-y+3z=7$.

**(b)** $-2x-y+3z=-7$.

**(c)** $x+2y+4z=11$.

**(d)** $x+5y=7$.

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**3**. The **angle** $\theta$ between the two planes $4x+y-z=1$ and $x-2y+2z=2$ is:

**(a)** $\theta=\frac{\pi}{2}$.

**(b)** $\theta=\cos^{-1}\sqrt{\frac{8}{153}}$.

**(c)** $\theta=\cos^{-1}\left(\frac{8}{3\sqrt{17}}\right)$.

**(d)** $\theta=0$.

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**4**. The **area** of the <u>**parallelogram**</u> whose two adjacent sides are formed by the following vectors $\vec{u}=2\vec{i}+\vec{j}-2\vec{k}$ and $\vec{v}=5\vec{i}-2\vec{j}+\vec{k}$ is equal to:

**(a)** $\frac{\sqrt{154}}{2}$.

**(b)** $\frac{\sqrt{234}}{2}$.

**(c)** $\sqrt{234}$

**(d)** $\sqrt{154}$.

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**5**. The **distance** between the point $P=(1,-2,2)$ and the plane $3x+y-2z=4$ is equal to:

**(a)** $\frac{1}{\sqrt{14}}$.

**(b)** $\frac{1}{2}$.

**(c)** $0$.

**(d)** $\frac{7}{\sqrt{14}}$.

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**6**. The vectors $\vec{u}=\vec{i}+2\vec{j}+4\vec{k}$, $\vec{v}=-2\vec{i}-3\vec{j}$ and $\vec{w}=\vec{j}+\vec{k}$ are:

**(a)** coplanar

**(b)** not coplanar

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**Question II:** [9 marks]

Let $\vec{r}(t) = 3\vec{\imath} + \sin(t)\vec{\jmath} + \cos(t)\,\vec{k}$ be the position vector of a moving point $P$.Then answer the following:

(i) Sketch the curve determined by $\vec{r}(t)$, for $0 \leq t \leq 2\pi$ and indicate the orientation.

(ii) Calculate the integral $\vec{I} = \int_0^{\pi} \vec{r}(t)\ dt$.

(iii) Find the tangential $a_T$ and normal $a_N$ components of acceleration.

(iv) Calculate the curvature $K(t)$ of this curve at any point.

(v) Calculate the unit tangent vector $\vec{T}(t)$ and the principal unit normal $\vec{N}(t)$.

(vi) Compute the arc length of the curve for $0 \leq t \leq \pi$.

**<u>Question III:</u>** [6+8= 14 marks]

A. Consider the vector field

$$\vec{F}(x,y,z) = (3+y^2)\vec{\imath} + (2xy+z^3)\vec{\jmath} + (3yz^2+8z)\vec{k}.$$

Then answer the following:

(i) Find $curl\,\vec{F}$.

(ii) Deduce from (i) that $\int_C \vec{F}.d\vec{r}$ is independent of path.

(iii) Find a potential function $u$ for $\vec{F}, i.e., \nabla u = \vec{F}$.

(iv) Use (iii) to compute $\int_C \vec{F}.d\vec{r}$, where $C$ is any piecewise smooth curve with initial point $(2,2,0)$ and endpoint $(3,2,1)$.

B. **Verify Green's theorem** for the line integral

$$\oint_C (2x - xy)dx + (y + xy^2)dy,$$

where $C$ is the planar positively oriented circle $x^2 + y^2 = 1$.

**Question IV:** [4+4=8 marks]

A. Use the divergence theorem to find the flux $\iint_S \vec{F}.\vec{n}\, dS$, where the field $\vec{F}$ is given by

$$\vec{F}(x,y,z) = y^2\,\vec{\imath} - yz\vec{\jmath} + xz\,\vec{k},$$

And the surface $S$ is boundary of $\mathcal{U} = \{(x,y,z) \colon x^2 + y^2 \leq 9,\ \ 0 \leq\ z \leq 2\}$, oriented so that the normal vectors $\vec{n}$ point to the outside of it.

B. Use Stokes' theorem to calculate the line integral $\oint_C \; F.dr$, for the vector field $\vec{F}$ given by

$$\vec{F}(x,y,z) = -2y^2\vec{i} + x^2\vec{j} + z^3\vec{k},$$

and the surface $S$ is the portion of the plane $x + y + z = 1$ that lies in the first octant, oriented so that the normal vectors $\vec{n}$ are pointing upwards.

Good Luck 😊