SYLLABUS and EXERCISES

CHAPTER 16: PARTIAL DIFFERENTIATION

Section 16.1: Functions of Several Variables

Do the following problems from the book

1, 3, 5, 8, 9, 14, 21, 22, 23, 24, 37.

Section 16.2: Limits and Continuity

I- Do the following problems from the book

3, 5, 6, 9, 12, 14, 16, 19, 20, 25, 26, 28, 29, 37, 38, 42.

II- Find the following limits, if they exist:

(1) $\lim_{(x,y)\to(1,2)} \frac{xy-2x-y+2}{x^2+y^2-2x-2y+5}$ (2) $\lim_{(x,y)\to(1,1)} \frac{x-y^4}{x^3-y^4}$ (3) $\lim_{(x,y)\to(0,0)} \frac{3xy}{5x^4+2y^4}$

(4) $\lim_{(x,y)\to(0,0)} \frac{3x^3-2x^2y+3y^2x-2y^3}{x^2+y^2}$ (5) $\lim_{(x,y)\to(0,0)} \frac{10xy}{5x^3+2y^3}$ (6) $\lim_{(x,y)\to(0,1)} \frac{x+(y-1)^3}{\sqrt{x^2+(y-1)^2}}$

(7) $\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz+zx}{x^2+y^2+z^2}$ (8) $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^3+y^6}$ (9) $\lim_{(x,y,z)\to(0,0,0)} \frac{y^3+x^3 sinz^3}{x^2+y^2+z^2}$

(10) $\lim_{(x,y)\to(2,1)} \frac{(y-1)(x-2)^2}{(y-1)^{3+}(x-2)^3}$ (11) $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^4+y^2}$ (12) $\lim_{(x,y,z)\to(0,0,0)} \frac{xy^2}{x^2+y^2+z^2}$

(13) $\lim_{(x,y)\to(1,1)} \frac{3x^3+xy^2-3xy-y^3}{x^2-y^2}$ (14) $\lim_{(x,y,z)\to(0,0,0)} \begin{cases} \frac{x}{y^2}-\frac{x}{e^{y^2}} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$

III – Show that

(1) $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$ (2) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} = 0$

(3) $\lim_{(x,y,z)\to(0,0,0)} f(x,y,z) = 0$, If $f(x,y,z) = \begin{cases} \frac{3xyz}{x^2+y^2+z^2} & (x,y,z)\neq(0,0,0) \\ 0 & ((x,y,z)=(0,0,0)) \end{cases}$

III- Discuss the continuity of the following functions on their domain:

(1) $f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$

(2) $f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)\neq(0,0) \end{cases}$, at $(x,y)=(0,0)$

(3) $f(x,y,z) = \begin{cases} \frac{x^3+y^3+z^3}{x^2+y^2+z^2} & (x,y,z)\neq(0,0,0) \\ 0 & (x,y,z)=(0,0,0) \end{cases}$, at $(x,y,z)=(0,0,0)$

(4) $f(x,y,z)=\begin{cases}\frac{xz-y^2}{x^2+y^2+z^2} & (x,y,z)\neq(0,0,0)\\ 0 & \big((x,y,z)=(0,0,0)\big)\end{cases}$, at $(x,y,z)=(0,0,0)$

(5) $f(x,y)=\begin{cases}\frac{xy}{|x|+|y|} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$

(6) $f(x,y,z)=\begin{cases}\frac{xzy^2}{x^2+y^2+z^2} & (x,y,z)\neq(0,0,0)\\ 0 & \big((x,y,z)=(0,0,0)\big)\end{cases}$

IV - Discuss the continuity of the following functions:

(1) $h(x,y)=e^{x^2+5xy+y^2}$ (2) $h(x,y)=sin\left(\sqrt{y-4x^2}\right)$ (3) $f(x,y)=e^{xy}sin^{-1}(x^2+y^2)$

(3) $h(x,y,z)=ln(36-4x^2-y^2-9z^2)$ (4) $f(x,y,z)=\frac{xz}{\sqrt{x^2+y^2+z^2-1}}$

Section 16.3: Partial Derivatives

I- Do the following problems from the book

4, 6, 8, 12, 13, 16, 17, 21, 22, 27, 29, 32, 34, 36, 39, 42, 47.

II- Do the following problems

1. Using the definition, find f_x, f_y of the function $f(x,y)=3x^2-2xy+y^2$

II. Let$f(x,y)=e^{x-y}sin(x+y)$, show that $(f_x)^2+\left(f_y\right)^2=\frac{2\left(f(x,y)\right)^2}{sin^2(x+y)}$

Section 16.4: Increments and Differentials

Do the following problems from the book

2, 9, 11, 12, 16, 18, 20, 39, 41.

1. Use the differential to approximate the change in the function

$w=f(x,y)=x^2ln(z^2+y^2)$, as (x, y, z) changes from (1, 2, 3) to (0.9, 1.9, 3.1).

2. Use the differential to approximate the change in the function

$w=f(x,y)=yx^{\frac{2}{5}}+\sqrt{x-y}$. As (x, y) changes from (52, 16) to (35, 18).

3. $f(x,y,z)=\begin{cases}\frac{xy^2z}{x^4+y^4+z^4} & (x,y,z)\neq(0,0,0)\\ 0 & \big((x,y,z)=(0,0,0)\big)\end{cases}$

(1) Show that $f_x(0,0,0)$, $f_y(0,0,0)$and $f_z(0,0,0)$ exist.

(2) Discuss the differentiability of $f(x,y,z)$ at $(0,0,0)$.

Section 16.5: Chain Rules

Do the following problems from the book

2, 4, 6, 10, 12, 14, 18, 19, 22, 33, 37, 38, 41, 42.

1. If $w = f(x,y,z) = x^2 + y^2 + z^2$, where $x = rcos\theta, y = rsin\theta$ and $z = r$. Use the differential to show that $dw = 4rdr$.

2. Let $z = f(x,y)$ be determined implicitly by $x^2 + z^2 + cos(xyz) - 4 = 0$. Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$. Then show that $2y\frac{\partial z}{\partial y} - x\frac{\partial z}{\partial x} = \frac{xyzsin(xyz)}{2z - xysin(xyz)}$

Section 16.8: Extrema of Functions of Several Variables

Do the following problems from the book;

1, 9, 11, 15, 20, 21, 23, 24, 26,29,31,32.

Section 16.9: Lagrange Multipliers

Do the following problems from the book; 1, 2, 3, 11.

Chapter 17: Multiple Integral

Section 17.1: Double Integral

Do the following problems from the book;

1 - 10, 13, 16, 18, 19, 20, 21, 23, 25, 26, 27, 29, 31, 32, 33, 37, 38, 39, 43, 44, 50.

1. Sketch the region bounded by the graphs of the given equations, and then evaluates the given integral $y = x, y = \sqrt{x}, x = 0; \iint_R siny^2\, dA$

2. Evaluate the double integral $\int_0^2 \int_{y/2}^1 e^{x^2}\, dx\, dy$

Section 17.2: Area and Volume

Do the following problems from the book;

2, 4, 6, 7, 11, 14, 18, 22, 24, 27, 28, 30, 31, 32.

1. Sketch the region bounded by the graphs of the equation $y = sinx$, $y = cosx, x = 0, x = \frac{\pi}{4}$. Then use the double integral to find its area.

Section 17.3: Double Integral by Polar Coordinate

Do the following problems from the book;

1 - 12, 13, 15, 17, 18, 19, 21, 23, 24.

1. Use polar coordinate to evaluate the double integral $\int_{-3}^{3} \int_0^{\sqrt{9-x^2}} (x^2 + y^2)^{3/2}\, dy\, dx$

Section 17.5: Triple Integral

Do the following problems from the book;

2, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 23, 26, 28.

Section17.7: Cylindrical Coordinates
2, 6, 7, 13 18, 20, 23,30(a), 32(just find the volume of the solid), 39, 40.

Section 17.8: Spherical Coordinates
2, 3, 11, 14, 25, 28, 32(just find the volume of the solid), 35,36, 39, 40.

CHAPTER 11: INFINITE SERIES

Section 11.1: Sequences

Do the following problems from the book;

3, 5, 7, 11, 12, 13, 16, 17, 18, 20, 22, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42.

Section 11.2: Convergent or Divergent Series

Do the following problems from the book;
2, 4, 5, 6, 8, 10, 14, 15, 18, 20, 25, 28, 30, 34, 37, 38, 39, 40, 42, 43, 45, 46, 50, 57, 58.

Section 11.3: Positive term Series

Do the following problems from the book;

2 - 11, 14, 15, 16, 18, 20, 22, 23, 24, 25, 26, 30, 31, 33, 34, 35, 39, 40, 42, 43, 45, 46, 57, 58.

Section 11.4: The Ratio and Root Test

Do the following problems from the book;

2, 4, 6, 8, 9, 10, 11, 12, 14, 15, 16, 18, 20, 21, 22, 23, 25, 27, 28, 29, 31, 32, 33, 34, 35, 38.

Section 11.5: Alternating Series and Absolute Convergence

Do the following problems from the book;

2 - 7, 9, 10, 12, 13, 14, 16, 19, 20, 21, 22, 27, 28, 29, 32, 33, 34, 35, 38, 40, 41, 43, 44, 45, 46.

Section 11.6: Power Series

Do the following problems from the book;

5, 6, 7, 8, 14, 15, 19, 23, 25, 27, 30, 35, 36, 41, 42, 44, 45, 46.

Section 11.7: Power Series Representation of Functions

Do the following problems from the book;

2, 4, 6, 7, 10, 13, 14, 16, 19, 22, 25, 29, 30, 32, 33, 34, 37.

Section 11.8: Maclaurin and Taylor Series

Do the following problems from the book;

2, 4, 8, 10, 13, 15, 18, 19, 21, 26, 29, 32, 34, 36, 38, 39, 42.
