

Question	I	II	III	IV	Total
Mark					

Question I: (0.5×8=4points)

Question	1	2	3	4	5	6	7	8
Answer								

Choose the correct answer, then fill in the table above:

(1) If a relation R is defined on \mathbb{Z} , by $aRb \leftrightarrow a + b$ is prime, then the following pair belongs to R

- (a) (1,3).
- (b) (3,4).
- (c) (2,7).
- (d) None of the previous.

(2) If $R = \{(a, b) | a > b^2\}$ is a relation defined on the set of real numbers, then $\bar{R} =$

- (a) $\{(a, b) | a \leq b^2\}$.
- (b) $\{(a, b) | a > b^2\}$.
- (c) $\{(a, b) | a^2 \leq b\}$.
- (d) None of the previous.

(3) In the set of positive integers with division relation $(\mathbb{Z}^+, |)$, the following pairs are comparable:

- (a) 2, 31.
- (b) 21, 14.
- (c) 4, 12.
- (d) None of the previous.

(4) If R and S are relations on a set A represented by the matrices

$$M_R = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \text{ then } M_{S \circ R} =$$

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$.

(b) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.

(c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

(d) None of the previous.

(5) Let $A = \{a, b, c\}$ and the relation R on A be represented by the matrix

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Then the relation R is

- (a) Reflexive.
- (b) Antisymmetric.
- (c) Symmetric.
- (d) None of the previous.

(6) Let $A = \{1, 2, 3, 4\}$. The equivalence relation produced by the partition $A_1 = \{1, 4\}$, $A_2 = \{2\}$, $A_3 = \{3\}$ is given by:

- (a) $\{(1,1), (1,4), (2,2), (3,3), (4,4)\}$.
- (b) $\{(1,1), (1,4), (4,1), (2,2), (3,3), (4,4)\}$.
- (c) $\{(1,1), (1,4), (4,1), (2,2), (2,3), (3,2), (3,3), (4,4)\}$.
- (d) None of the previous.

(7) Let $G = (V, E)$ be an r – regular graph with $|V| = |E|$. Then the value of r is

- (a) 16.
- (b) 4.
- (c) 2.
- (d) None of the previous.

(8) Let H be a graph with 8 edges and degree sequence $1, 3, x, x$, then $x =$

- (a) 6.
- (b) 2.
- (c) 1.
- (d) None of the previous.

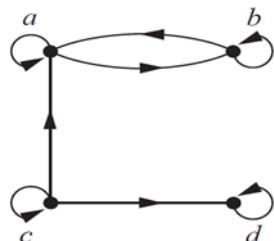
Question II: (2+4=6 points)

A. For the partial ordered set on $A = \{1, 2, 3, 4\}$, given by:

$$S = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\},$$

draw the Hasse diagram representing the relation S. Is the poset a totally ordered set? Justify your answer.

B. Let R be a relation on $A = \{a, b, c, d\}$, represented by the directed graph (digraph) below:



Then answer the following:

- (i) List all ordered pairs of R .
- (ii) Is the relation R reflexive? Justify your answer.
- (iii) Is the relation R symmetric? Justify your answer.
- (iv) Is the relation R antisymmetric? Justify your answer.

Question III: (4.5+1.5+1+1=8 points)

Let R be a relation defined on the set of integers \mathbb{Z} by $R = \{(a, b) : a - b \text{ even}\}$. Then answer the following:

(i) Prove that R is an equivalence relation on \mathbb{Z} .

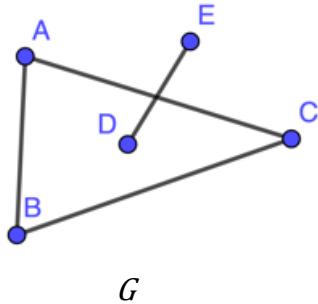
(ii) Find the equivalence classes $[0]$ and $[1]$.

(iii) Use (ii) to find a partition of \mathbb{Z} .

(iv) Is the relation R a partial ordered set on \mathbb{Z} ? Justify your answer.

Question IV: ($2.5+3+1.5=7$ points)

A. Answer the following questions about the graph G :



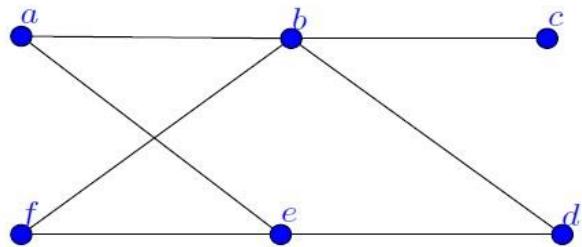
(i) Is the graph G connected? Justify your answer.

(ii) How many connected components are in the graph G ?

(iii) Find a simple circuit. What is its length?

(iv) What is the degree of C .

B. Answer the following questions about the graph J below

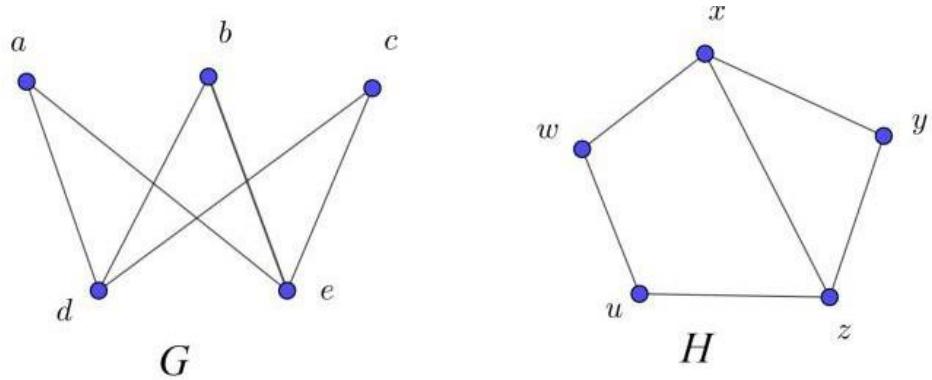


J

(i) Is the graph J bipartite? Justify your answer.

(ii) Represent the graph J in an **adjacency matrix**.

C. Are the graphs G and H below isomorphic? Justify your answer.



Good Luck 😊

