

Question	I	II	III	IV	Total
Mark					

**Question I:** (0.5×8=4points)

Question	1	2	3	4	5	6	7	8
Answer								

Choose the correct answer, then fill in the table above:

(1) If a relation  $R$  is defined on  $\mathbb{Z}$ , by  $aRb \leftrightarrow a + b$  is prime, then the following pair belongs to  $R$

- (a) (1,3).
  - (b) (3,4).
  - (c) (2,7).
  - (d) None of the previous.
- 

(2) If  $R = \{(a, b) | a > b^2\}$  is a relation defined on the set on real number, then  $\bar{R} =$

- (a)  $\{(a, b) | a \leq b^2\}$ .
  - (b)  $\{(a, b) | a > b^2\}$ .
  - (c)  $\{(a, b) | a^2 \leq b\}$ .
  - (d) None of the previous.
- 

(3) In the set of positive integers with division relation  $(\mathbb{Z}^+, |)$ , the following pairs are comparable:

- (a) 2, 31.
  - (b) 21, 14.
  - (c) 4, 12.
  - (d) None of the previous.
- 

(4) If  $R$  and  $S$  are relations on a set  $A$  represented by the matrices

$$M_R = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \text{ then } M_{S \circ R} =$$

- (a)  $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ .
- (b)  $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ .
- (c)  $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ .

(d) None of the previous.

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(5) Let  $A = \{a, b, c\}$  and the relation  $R$  on  $A$  be represented by the matrix

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Then the relation  $R$  is

- (a) Reflexive.
  - (b) Antisymmetric.
  - (c) Symmetric.
  - (d) None of the previous.
- 

(6) Let  $A = \{1, 2, 3, 4\}$ . The equivalence relation produced by the partition  $A_1 = \{1, 4\}$ ,  $A_2 = \{2\}$ ,  $A_3 = \{3\}$  is given by:

- (a)  $\{(1,1), (1,4), (2,2), (3,3), (4,4)\}$ .
  - (b)  $\{(1,1), (1,4), (4,1), (2,2), (3,3), (4,4)\}$ .
  - (c)  $\{(1,1), (1,4), (4,1), (2,2), (2,3), (3,2), (3,3), (4,4)\}$ .
  - (d) None of the previous.
- 

(7) Let  $G = (V, E)$  be an  $r$ -regular graph with  $|V| = |E|$ . Then the value of  $r$  is

- (a) 16.
  - (b) 4.
  - (c) 2.
  - (d) None of the previous.
- 

(8) Let  $H$  be a graph with 8 edges and degree sequence  $1, 3, x, x$ , then  $x =$

- (a) 6.
  - (b) 2.
  - (c) 1.
  - (d) None of the previous.
-

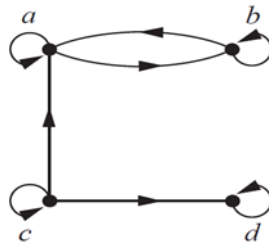
**Question II:** (2+4=6 points)

A. For the partial ordered set on  $A = \{1, 2, 3, 4\}$ , given by:

$$S = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\},$$

draw the **Hasse diagram** representing the relation  $S$ . Is the poset a totally ordered set? Justify your answer.

B. Let  $R$  be a relation on  $A = \{a, b, c, d\}$ , represented by the directed graph (digraph) below:



Then answer the following:

(i) List all ordered pairs of  $R$ .

(ii) Is the relation  $R$  reflexive? Justify your answer.

(iii) Is the relation  $R$  symmetric? Justify your answer.

(iv) Is the relation  $R$  antisymmetric? Justify your answer.

**Question III:** (4.5+1.5+1+1=8 points)

Let  $R$  be a relation defined on the set of integers  $\mathbb{Z}$  by  $R = \{(a, b) : a - b \text{ even}\}$ . Then answer the following:

(i) Prove that  $R$  is an equivalence relation on  $\mathbb{Z}$ .

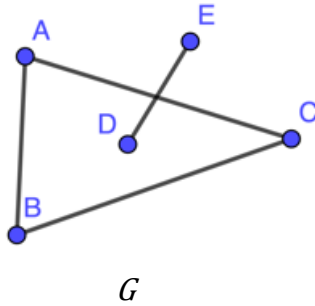
(ii) Find the equivalence classes  $[0]$  and  $[1]$ .

(iii) Use (ii) to find a partition of  $\mathbb{Z}$ .

(iv) Is the relation  $R$  a partial ordered set on  $\mathbb{Z}$ ? Justify your answer.

**Question IV:** ( 2.5+3+1.5=7points)

A. Answer the following questions about the graph  $G$ :



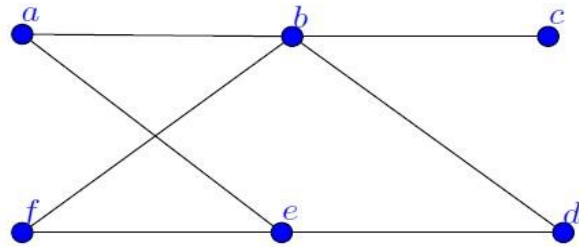
(i) Is the graph  $G$  connected? Justify your answer.

(ii) How many connected components are in the graph  $G$ ?

(iii) Find a simple circuit. What is its length?

(iv) What is the degree of  $C$ .

B. Answer the following questions about the graph  $J$  below

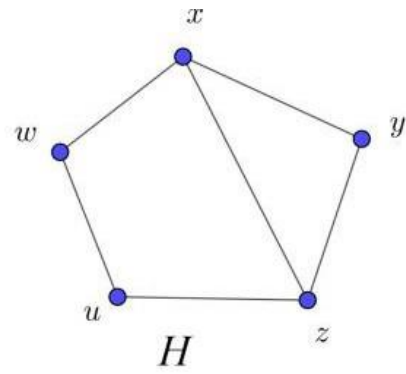
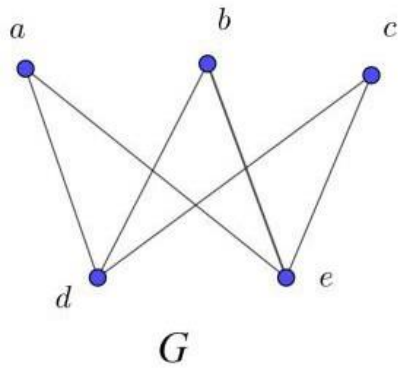


$J$

(i) Is the graph  $J$  bipartite? Justify your answer.

(ii) Represent the graph  $J$  in an **adjacency matrix**.

C. Are the graphs  $G$  and  $H$  below isomorphic? Justify your answer.



Good Luck 😊

