Question	I	II	III	IV	Total
Mark					

<u>Question I</u>: (0.5×8=4points)

Question	1	2	3	4	5	6	7	8
Answer	a	b	a	b	c	c	c	a

Choose the correct answer, then fill in the table above:

(1) If R is a symmetric relation on a set A, then the inverse relation, R^{-1}, is also symmetric
(a) True
(b) False

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(2) Let $R = \{(x, y): y = 2x + 1\}$, and $S = \{(x, y): y = x^2\}$. Where R and S are defined on $\mathbb{Z}$, then $R \circ S$ is defined as
(a) $\{(x, y): y = (2x + 1)^2\}$.
(b) $\{(x, y): y = 2x^2 + 1\}$.
(c) $\{(x, y): y = (2x + 1)x^2\}$.
(d) None of the previous.

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(3) In the set of positive integers with division relation $(\mathbb{Z}^+, |)$, 4 and 8 are:
(a) comparable
(b) incomparable

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(4) If R_1 *and* R_2 are relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \text{ then } M_{R_1 \cap R_2} =$$

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(d) None of the previous

(5) For the equivalence relation on $\mathbb{Z}$ defined by $aRb \leftrightarrow a \equiv b\ (mod 3)$, $\mathbb{Z}$ can be partitioned

(a) $\{[0],[1],[3]\}$.

(b) $\{[-1],[0],[1],[2]\}$.

(c) $\{[-1],[0],[1]\}$.

(d) None of the previous.

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(6) The number of edges of the $20-regular$ graph with 10 vertices graph is:

(a) 180.

(b) 90.

(c) 100.

(d) None of the previous.

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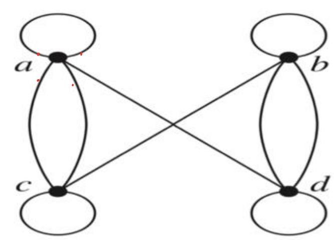
(7) For the graph below, the degree of the vertex a is

(a) 4.

(b) 3.

(c) 5.

(d) None of the previous.



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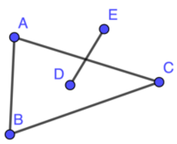
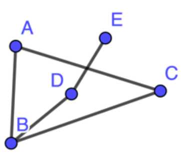
(8) Which of the following graphs is a **disconnected** graph?

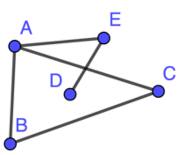
(a)



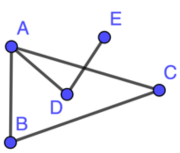
(b)



(c)



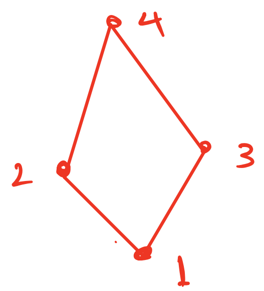
(d)



Question II: (1.5+5.5=7points)

A. Draw the Hasse diagram of the poset on $A = \{1, 2, 3, 4\}$, given by:

$$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (3,4), (4,4)\}.$$



B. Let R be a relation on $A = \{1, 2, 3, 4\}$, defined by

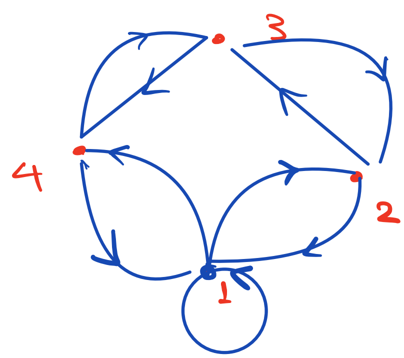
$$aRb \leftrightarrow a + b \text{ is prime}.$$

Then answer the following:

(i) List all ordered pairs of R.

$\{(1,1), (1,2), (2,1), (3,2), (2,3), (4,1), (1,4), (4,3), (3,4)\}$

(ii) Draw a directed graph (digraph) representing the relation R .



(iii) Is the relation reflexive? Justify your answer.

No, because $(4,4) \notin R$

(iv) Is the relation antisymmetric? Justify your answer.

No, because $(1,2), (2,1) \in R$ but $1 \neq 2$

Question III: (4.5+1.5+1=7points)
Let R be a relation defined on the set of integers $\mathbb{Z}$ by $R = \{(a,b) : a^2 = b^2\}$. Then answer the following:

(i) Prove that R is an equivalence relation on $\mathbb{Z}$.

1) R is reflexive as $\forall a \in \mathbb{Z}$, $a^2 = a^2$

1.5

$\Rightarrow (a,a) \in R$

2) R is symetric :- $\forall a,b \in \mathbb{Z}$

1.5

$aRb \Leftrightarrow a^2 = b^2 \Leftrightarrow b^2 = a^2 \Leftrightarrow bRa$

3) R is transitive

1.5

$\forall a,b,c \in \mathbb{Z}$

let $aRb \wedge bRc \Rightarrow a^2 = b^2 \wedge b^2 = c^2$

$\Rightarrow a^2 = c^2 \Longrightarrow aRc$.

From ①, ② and ③ R is an equivalence relation.

(ii) Find the equivalence classes $[x]$ and $[5]$.

$[x] = \{y \in \mathbb{Z} : xRy\} = \{y \in \mathbb{Z} : x^2 = y^2\} = \{\mp x\}$

$[5] = \{5, -5\}$

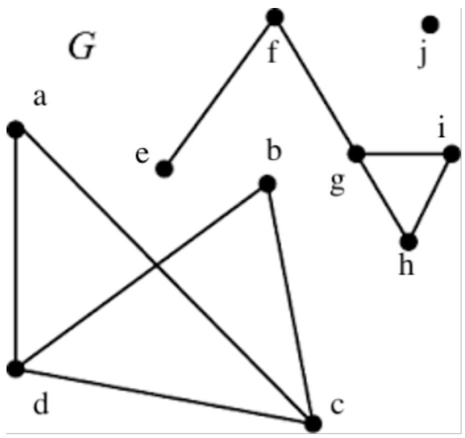
(iii) Is the relation a poset? Justify your answer.

No, because it is not antisymetric

$1 R -1$ and $-1 R 1$ but $1 \neq -1$.

Question IV: (2.5+3+1.5=7points)

A. Answer the following questions about the graph G:



(i) Is the graph G connected? Justify your answer.

No, we cant find pat between "j" and "a".

(ii) How many connected components are in the graph G.

3

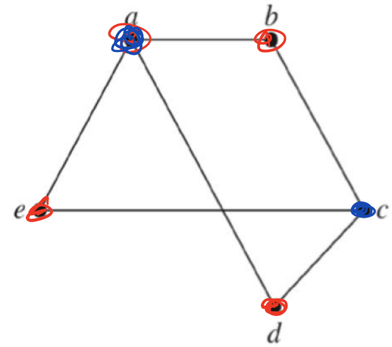
(iii) Find a path from i to e of length 4.

i, h, g, f, e

(iv) Does the graph have an isolated vertex? If so, name it.

Yes, The vertex j.

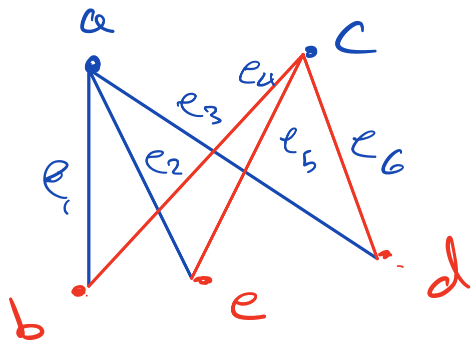
B. Answer the following questions about the graph J below



J

(i) Is the graph J bipartite? Justify your answer.

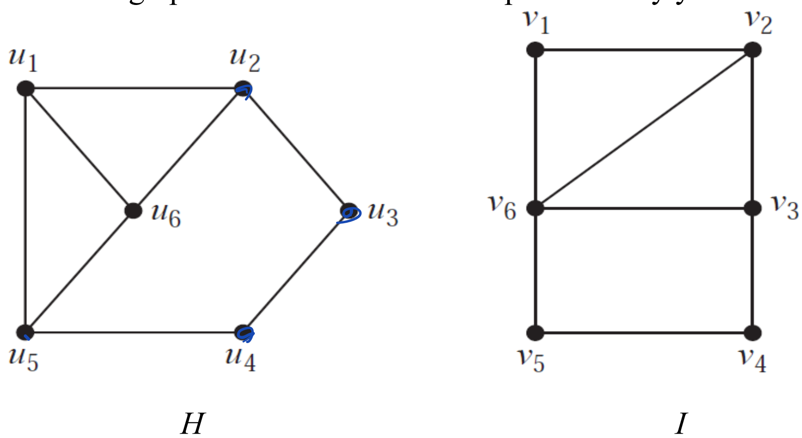
Yes,



(ii) Represent the graph J in an adjacency matrix.

$$
\begin{array}{c}
\\ a \\ b \\ c \\ d \\ e
\end{array}
\begin{array}{c}
\begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

C. Are the graphs H and I below isomorphic? Justify your answer.



No, I has a vertix of degree 4 while H dosn't.

H has 4 vertixes of degree 3 while I dosnt,

Good Luck 😊