Questions	Q1	Q2	Q3	Q4	Total
Marks					

Question 1: (8 points)

Question Number	1	2	3	4	5	6	7	8
Answer	a	b	c	b	d	b	c	b

Choose the correct answer, then fill it in the above table:

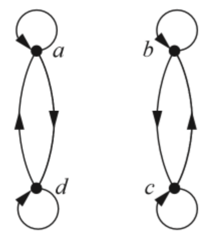
1) In the poset $(P(\mathbb{N}), \subseteq)$, the sets $\{1,2\}$ and $\{1,2,3\}$ are
   (a) Comparable.
   (b) Incomparable.

2) The relation $\{(a, b) | a + b \text{ is odd}\}$ on $\mathbb{Z}$ is a partial ordering.
   (a) True.
   (b) False.

3) Let $R = \{(a, b) | a \equiv b(mod3)\}$. Then
   (a) $[3] = [5]$ (b) $[1] = [3]$ (c) $[2] = [-1]$ (d) None.

4) If $R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$ is a relation on $\{1,2,3,4\}$ then the partition formed by this partition is:
   (a) $\{\{1,2\}, \{3,4\}\}$
   (b) $\{\{1,2\}, \{3\}, \{4\}\}$
   (c) $\{\{1\}, \{2\}, \{3,4\}\}$
   (d) None.

5) The directed graph below represents a relation R that is:



(a) Reflexive, symmetric, antisymmetric, and transitive.
(b) Reflexive, not symmetric, antisymmetric, and not transitive.
(c) Reflexive, not symmetric, not antisymmetric, but transitive.
(d) Reflexive, symmetric, not antisymmetric, and transitive.

6) The graph $K_{3,3}$ is considered
(a) 2-regular.
(b) 3-regular.
(c) 9-regular.
(d) None.

$(b, a) \mid a \leq b$

7) If $R = \{(a, b) | a \leq b\}$, is a relation on $\mathbb{R}$, then R^{-1} equal
(a) $\{(a, b) | a < b\}$.
(b) $\{(a, b) | a > b\}$
(c) $\{(a, b) | a \geq b\}$
(d) None

8) If $R = \{(x, y) | y = \sqrt{x^2 + 1}\}$ and $S = \{(x, y) | y = \sqrt{x}\}$, where R and S defined on $\mathbb{R}^+$, then $R \circ S$ is defined as
(a) $\{(x, y) | y = \sqrt{\sqrt{x^2 + 1}}\}$.
(b) $\{(x, y) | y = \sqrt{x + 1}\}$
(c) $\{(x, y) | y = \sqrt{\sqrt{x} + 1}\}$
(d) None.

Question 2: (4+5 points)

(A) Let R and S be relations on $\{1,2,3\}$ and let $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ be the matrices of R and S respectively.

(1) List the ordered pairs of R and S.

$R = \{(1,1), (1,3), (2,1), (2,2), (3,3)\}$

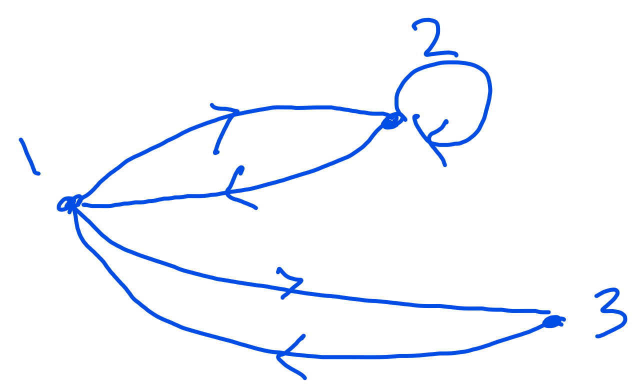
$S = \{(1,2), (1,3), (2,1), (2,2), (3,1)\}$

(2) Find $M_{R\cap S} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(3) Find $M_{R\circ S} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(4) Find $M_{\bar{R}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(5) Represent S using directed graph.



(B) Let R be the relation defined on $\mathbb{Z}$ by

$$aRb \Leftrightarrow 3a \equiv b(mod2)$$

(1) Show that R is an equivalence relation.

R is reflexive: as $2 | 3a - a$ i.e $2 | 2a$ $\therefore$ aRa $\forall a \in \mathbb{Z}$

R is symmetric: if $aRb \rightarrow 2 | 3a - b$
$\rightarrow 3a - b = 2K$ for some $K \in \mathbb{Z}$

$3b - a = 3(3a - 2K) - a$
$= 8a - 6k = 2(4a - 3k)$

$\therefore$ $2 | 3b - a$ $\therefore$ bRa

R is transitive: if $aRb \wedge bRc$
$\rightarrow 2 | 3a - b \wedge 2 | 3b - c \rightarrow 3a - b = 2k, 3b - c = 2L$ $K, L \in \mathbb{Z}$

$\rightarrow 3a - c = b + 2k + 2L - 3b = 2k + 2L - 2b$
$= 2(K + L - b)$

$\therefore$ $2 | 3a - c$ and aRc.

(2) Find [1] and [2]

$[1] = \{a | 1Ra\} = \{a | 2 | 3 - a\} = \{a | 3 - a \text{ is even}\}$
$= \{ -3, -1, 1, 3, 5 \}$

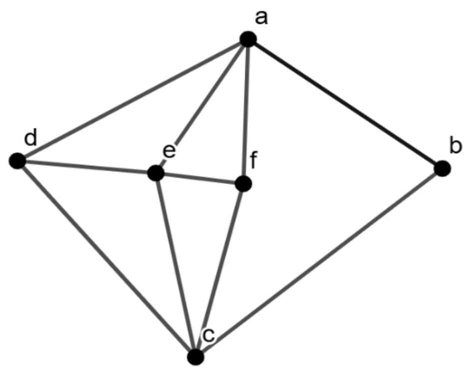
$[2] = \{a | 2Ra\} = \{a | 2 | 6 - a\} = \{\ldots, -2, 0, 2, 4, \ldots\}$

(3) Is $[4] \cap [3] = \emptyset$? Justify your answer

yes, as $3 \not{R} 4$

<u>Question 3:</u> (4+2 points)

(A) Consider the graph G below



(1) Find deg(a) and deg(d).

$\deg(a) = 4$

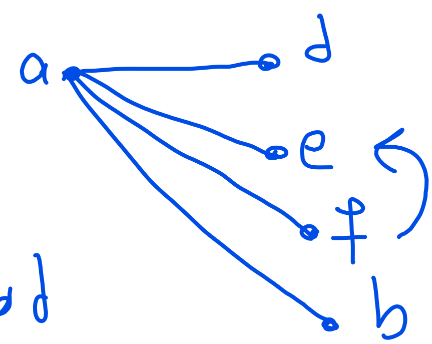
$\deg(d) = 3$

(2) Find $N(A)$ where $A = \{a,d\}$.

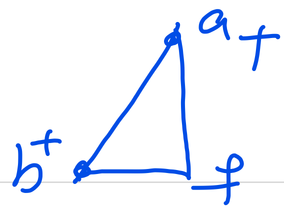
$N(A) = \{a, b, c, d, e, f\}$

(3) Is the graph bipartite? Justify your answer.

NO,



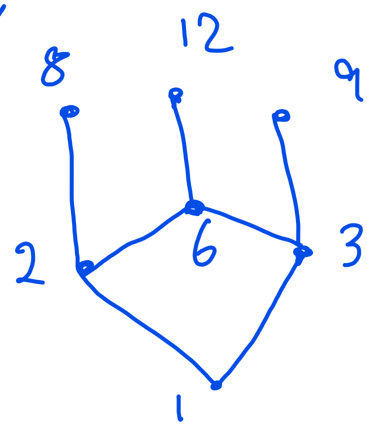
or by any other method



(B) Let $A = \{1,2,3,6,8,9,12\}$. Draw the Hasse diagram representing the poset $\{(a,b) | a \text{ divides } b\}$ on A. Is the poset a totally ordered set? Justify your answer.

The poset isn't totally ordered set because $2 \nmid 3$ and $3 \nmid 2$

(or not all the points are on the same line)



Question 4: (2 points)

For $K_{n,3}$ where $n > 3$, find the following:

(1) $|V|$ $= n+3$

(2) Degree sequence of vertices in $K_{n,3}$

$$n, n, n, \underbrace{3, 3, \ldots, 3}_{n\text{-times}}$$

(3) $|E|$ $= 3n$

(4) $|E|$ in $\overline{K_{n,3}}$

$$|E| = |E_{K_{3+n}}| - |E_{K_{n,3}}| = \frac{(n+3)(n+2)}{2} - 3n$$

OR: $$|E| = |E_{K_3}| + |E_{K_n}| = 3 + \frac{n(n-1)}{2}$$

دعواتي لكن بالتوفيق