College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

First Midterm Exam
Academic Year 1445 Hijri- SecondSemester

Exam Information معلومات الامتحان			
Course name	**Discrete Mathematics الرياضيات المحددة**		اسم المقرر
Course Code	**151 ريض**		رمز المقرر
Exam Date	**2024-03-06**	**1445-08-25**	تاريخ الامتحان
Exam Time	**10: 00 AM**		وقت الامتحان
Exam Duration	**2 hours**	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
- Calculators are not allowed.

تعليمات عامة:

- عدد صفحات الامتحان 6 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
- الآلة الحاسبة ممنوعة.

هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	C.L.O 1.1	I		
2	C.L.O 2.1	II+ III+IV		
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Question	I	II	III	IV	Total
Mark					

Question I: (8points)

Question	1	2	3	4	5	6	7	8
Answer								

Choose the correct answer, then fill in the table above:

(1) If $\boldsymbol{p \to q}$ **i**s false, then $\neg \boldsymbol{p} \vee \boldsymbol{q}$ is

(a) True.
(b) False.

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(2) The proposition " $\boldsymbol{x^2 \leq x \; whenever \; 0 \leq x \leq 1}$" is equivalent to:

(a) $\boldsymbol{x^2 \leq x \; implies \; 0 \leq x \leq 1.}$
(b) $\boldsymbol{x^2 \leq x \;\; if \; and \; only \; if \;\; 0 \leq x \leq 1}$.
(c) **If** $\boldsymbol{0 \leq x \leq 1}$ **then** $\boldsymbol{x^2 \leq x}$.
(d) None of the previous.

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(3) The proposition $\neg \, \exists \boldsymbol{x}: (\boldsymbol{x^3 = 1 \; \wedge 2x = 1})$ is equivalent to:
(a) $\exists \boldsymbol{x}: (\boldsymbol{x^3 \neq 1 \; \vee 2x \neq 1})$.
(b) $\forall \boldsymbol{x}: (\boldsymbol{x^3 \neq 1 \; \wedge 2x \neq 1})$.
(c) $\forall \boldsymbol{x}: (\boldsymbol{x^3 \neq 1 \; \vee 2x \neq 1})$.
(d) None of the previous.

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(4) The proposition "$\neg[\boldsymbol{p} \vee \neg \boldsymbol{p}] \; \to \boldsymbol{q}$" is a

(a) Tautology.
(b) Contradiction.
(c) Contingency.

(5) The truth value of the statement $\exists! \boldsymbol{x} \in \mathbb{R}$ $\textbf{such that } \boldsymbol{x^2 - 5 = 0}$ is:

(a) True.

(b) False.

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(6) The inverse of the proposition $\boldsymbol{if\ x + y = 2\ then\ 2x + 2y > 2}$" is:

(a) **if** $\boldsymbol{x + y \neq 2\ then\ 2x + 2y \leq 2}$.

(b) **if** $\boldsymbol{2x + 2y > 2\ then\ x + y = 2.}$

(c) **if** $\boldsymbol{2x + 2y \leq 2\ then\ x + y \neq 2.}$

(d) None of the previous.

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(7) If the domain of the propositional function $P(x)$ consists of the integers $1, 2, 3\ and\ 4$, then the following is equivalent to the statement $\neg\forall \boldsymbol{x P(x)}$:

(a) $\neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$.

(b) $\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$.

(c) $\mathrm{P}(1) \vee P(2) \vee P(3) \vee P(4)$.

(d) None of the previous.

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(8) Let $P(x, y)$ be the statement " $\boldsymbol{y - 2x\ is\ \ odd}$ " then the following is true:

(a) $P(1,5)$.

(b) $P(1,2)$.

(c) $P(3,2)$.

(d) None of the previous.

Question II: (2+2+2=6 points)

A. **Without using truth tables** prove the following:

$$\neg \boldsymbol{p} \to (\neg \mathbf{p} \wedge \mathbf{q}) \equiv \neg \boldsymbol{p} \to \boldsymbol{q}.$$

B. Prove that: $\mathbf{2^{\boldsymbol{n}} + 1} \geq \boldsymbol{n^2}$, for all nonnegative integers n less than 4.

C. Prove by **contradiction**: "$If\ m \neq 0\ is\ rational\ and\ n\ is\ irrational\ then\ mn\ is\ irrational$".

<u>Question III</u>: (1+5=6 points)

A. Prove that the statement "$for\ every\ n \in \mathbb{Z}$, $\boldsymbol{n^2 > 2n}$" is **false**.

B. For every integer n, prove that: “ $\boldsymbol{n}$ is even if and only if $\boldsymbol{3n^2 + 5}$ is odd”.

<u>Question IV:</u> (5 points)

Let $\{a_n\}$ be a sequence defined inductively as:

$$a_0 = 0, \qquad a_1 = 1, \;\; a_n = 3a_{n-1} - 2a_{n-2} \;, \;\; \forall\, n \geq 2.$$

Using **<u>Strong Induction</u>** prove that:

$$\boldsymbol{a_n = 2^n - 1}, \;\; for\ all\ nonnegative\ integers\ n.$$

Good Luck 😊