Questions	Q1	Q2	Q3	Q4	Q5	Total
Marks						

Question 1: (8 points)

Question Number	1	2	3	4	5	6	7	8
Answer								

Choose the correct answer, then fill it in the above table:

1) The truth value of "$\forall x \in \mathbb{R}$, if $x^2 \geq 1$, then $x > 0$" is:
   (a) True.
   (b) False.

2) The proposition $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$ for any prpositions p and q is a:
   (a) Tautology.
   (b) Contradiction.
   (c) Contingency

$(p \to q) \wedge \neg(p \to q)$

3) The proposition $(p \to q) \wedge (p \wedge \neg q)$ is logically equivalent to:
   (a) **T** (b) **F** (c) p (d) q

4) The statement $p \to q$ is equivalent to:
   (a) $\neg p \to q$
   (b) $p \to \neg q$
   (c) $\neg q \to \neg p$
   (d) $\neg q \to p$

5) The negation of the statement $[\exists x(x^2 > x)]$ and $[\forall x(x^2 \neq 1)]$ is

(a) $[\forall x(x^2 \geq x)]$ or $[\exists x(x^2 = 1)]$

(b) $[\exists x(x^2 > x)]$ or $[\forall x(x^2 = 1)]$

(c) $[\forall x(x^2 \leq x)]$ or $[\exists x(x^2 = 1)]$

(d) $[\exists x(x^2 \leq x)]$ or $[\forall x(x^2 = 1)]$

6) The statement "$x - 3 < 0$" is true when the domain is the set of:

(a) All positive real numbers.

(b) All negative real numbers.

(c) All real numbers.

(d) All real numbers except $x = 3$.

7) Let $P(x, y)$ be the statement: "$y - 1 = x$". Then

(a) $P(2,3)$ is false.

(b) $P(3,2)$ is true.

(c) $P(5,4)$ is false

(d) $P(4,5)$ is false

8) Let $p(x)$ be the statement "$x^2 > 11$" and the universe of discourse consists of the positive integers not exceeding 4, then the truth value of $\exists x P(x)$ is equal to

(a) $\big(P(1) \vee P(2)\big) \wedge (P(3) \wedge P(4))$.

(b) $P(1) \vee P(2) \vee P(3) \vee P(4)$.

(c) $P(1) \wedge P(2) \wedge P(3) \wedge P(4)$.

(d) $\big(P(1) \vee P(2)\big) \vee (P(3) \wedge P(4))$.

Question 2: (3+2.5 points)

(A) Consider the proposition: "If $2 + 6 = 3$ then $\sqrt{2}$ *is* irrational ". Find the following:

(1) The truth value of the proposition. True

(2) The convers and its truth value. $q \to p$

If $\sqrt{2}$ is irrational then $2+6=3$

False

(3) The inverse and its truth value. $\neg p \to \neg q$

If $2+6 \neq 3$ then $\sqrt{2}$ is not irrational

False

(4) The contraposition and its truth value.

If $\sqrt{2}$ is not rational then $2+6 \neq 3$

True

(B) Without using truth tables show that $(p \wedge q) \to r \equiv (p \to r) \vee (q \to r)$.

$$(p \wedge q) \to r \equiv \neg(p \wedge q) \vee r$$ $p \to q \equiv \neg p \vee q$

$$\equiv (\neg p \vee \neg q) \vee r$$ De Morgan's Law

$$\equiv \neg p \vee r \vee \neg q$$ commutativity

$$\equiv \neg p \vee r \vee r \vee \neg q$$ $r \vee r \equiv r$

$$\equiv (\neg p \vee r) \vee (\neg q \vee r)$$ commutativity and associativity

$$\equiv (p \to r) \vee (q \vee r)$$

Question 3: (2.5+2.5 points)

(A) **By contradiction:** Prove that if x is rational and y is irrational then $x + y$ is irrational.

Assume that x is rational and y is irrational but $x+y$ is rational. Then $x = \frac{p}{q}$ and $x+y = \frac{m}{n}$, where $p, q, m, n \in \mathbb{Z}$, $q \neq 0$, $n \neq 0$

So $y = (x+y) - x = \frac{m}{n} - \frac{p}{q} = \frac{mq - np}{nq}$, $mq-np, nq \in \mathbb{Z}$, $nq \neq 0$

This mean that y is rational, a contradiction.

(B) Prove that for all integer n, if $n + 1$ is even, then n^2 is odd.

Assume that $n+1$ is even, then $n+1 = 2k$ for some $k \in \mathbb{Z}$

then $n = 2k - 1$

so $n^2 = (2k-1)^2 = 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$, since $2k^2 + 2k \in \mathbb{Z}$

then n^2 is odd.

Question 4: (2.5+1 points)

(A) Prove that for all integer n, if $n^2 + n$ is odd, then n is odd.

By contraposition:

Assume that n is even

then $n = 2k$ for some $k \in \mathbb{Z}$

and $n^2 = 4k^2$

Hence $n^2 + n = 4k^2 + 2k$

$= 2(2k^2 + k)$, since $2k^2 + k \in \mathbb{Z}$

then $n^2 + n$ is even.

$\therefore$ if n^2+n is odd then n is odd.

(B) Show that that statement "For every positive integer n, $n^2 + 1 > 2n$" is false.

$n = 1$ is a counterexample as $1^2 + 1 > 2$ is false.

Question 5: (4 points)

The sequence $\{a_n\}$ is defined recursively by

$$a_1 = 1, a_2 = 4, a_n = 2a_{n-1} - a_{n-2}, n \geq 3$$

Use the Principal of Strong Mathematical induction to prove that $a_n = 3n - 2 \; \forall n \geq 1$.

Let $P(n)$: " $a_n = 3n - 2$ "

Basis step:

$P(1)$ $\quad$ $L.H.S = a_1 = 1$ $\quad$ $R.H.S = 3(1) - 2 = 1$

$\therefore$ $P(1)$ is true.

$P(2)$ $\quad$ $L.H.S = a_2 = 4,$ $\quad$ $R.H.S = 3(2) - 2 = 4$

$\therefore$ $P(2)$ is true.

Inductive step: Assume that $P(j)$ is true $\forall \; 1 \leq j \leq k$ (IH)

where k is an integer.

$$\begin{aligned} a_{k+1} &= 2a_k - a_{k-1} \\ &= 2(3k-2) - (3(k-1)-2) \\ &= 6k - 4 - 3k + 3 + 2 \\ &= 3k + 1 \\ &= 3k + 3 - 2 \\ &= 3(k+1) - 2 \end{aligned}$$

$\therefore$ $P(k+1)$ is true

By ①, ② $P(n)$ is true $\forall n \geq 1$.

دعواتي لكن بالتوفيق