College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Final Exam
Academic Year 1445-1446 Hijri- Second Semester

Exam Information معلومات الامتحان			
Course name	**Discrete Mathematics**		**اسم المقرر**
Course Code	**Math 151**		**رمز المقرر**
Exam Date	**2025-05-26**	**1446-11-28**	**تاريخ الامتحان**
Exam Time	**08: 00 AM**		**وقت الامتحان**
Exam Duration	**3 hours**	**ثلاث ساعات**	**مدة الامتحان**
Classroom No.			**رقم قاعة الاختبار**
Instructor Name	**د.جواهر المفرج**		**اسم استاذ المقرر**

Student Information معلومات الطالب		
Student's Name		**اسم الطالب**
ID number		**الرقم الجامعي**
Section No.		**رقم الشعبة**
Serial Number		**الرقم التسلسلي**

General Instructions:

- Your Exam consists of 8 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
- Calculators are not allowed

تعليمات عامة:

- عدد صفحات الامتحان 8 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
- لا يسمح بالالات الحاسبه

هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	CLO1.2	Q2a	4	
2	CLO2.1	Q1,Q4	9+8	
3	CLO2.2	Q2b	5	
4	CLO2.3	Q3	7	
5	CLO2.4	Q5	7	40
6				
7				
8				

EXAM COVER PAGE

Question	Q1	Q2	Q3	Q4	Q5	Total
Grade						

Q1. (a) Without using truth tables, show that $[(p \wedge \neg q) \vee \neg p] \rightarrow r$ is logically equivalent to $\neg r \rightarrow (p \wedge q)$. (3)

(b) Use induction to prove the following for every $n \geq 1$:

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \cdots + \frac{2n+1}{n^2(n+1)^2} = 1 - \frac{1}{(n+1)^2}. \ (4)$$

(c) Let a, b and c be real numbers. Use contraposition to show that if $2a - c > 7$, then $a - b > 1$ or $c - 2b < -5$. (2)

Q2. (a) Let R be the relation on $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$ defined by:

$$xRy \Leftrightarrow x - y = 10.$$

Determine whether R is reflexive, symmetric, antisymmetric or transitive. (4)

(b) Let E be the relation on $\mathbb{Z}$ defined by $mEn \Leftrightarrow 4 \mid (m^2 - n^2)$.

(i) Prove that E is an equivalence relation. (3)

(ii) Show that $[m] = [7m]$ for every $m \in \mathbb{Z}$. (1)

(iii) Show that $m + 1 \notin [m]$ for every $m \in \mathbb{Z}$. (1)

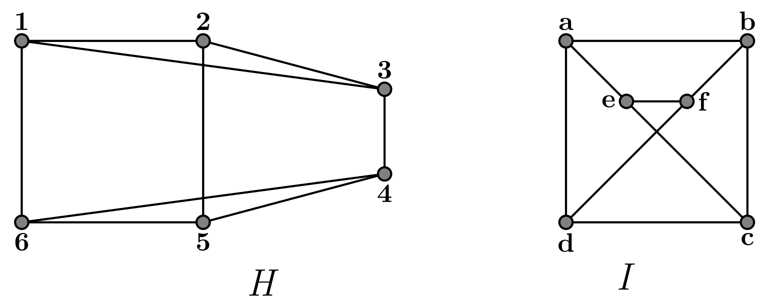
Q3. (a) Let G be a simple graph with degree sequence $x, x, x, x, x, x, 2x, 4x$ and having 6 edges.

(i) Find x. (1)

(ii) Show that G cannot be connected. (1)

(iii) Find the number of edges of the complement $\overline{G}$ of G. (1)

(b) Determine if the following graphs H and I are isomorphic. (2)



(c) Determine whether the graph I in **(b)** is bipartite, and if so, then find a bipartite representation. (2)

Q4.(a) (i) Give an example of a tree T, whose complement $\overline{T}$ is a tree. (1)

(ii) Give an example of a tree T, whose complement $\overline{T}$ is not a tree. (1)

(iii) Draw 3 (nonisomorphic) trees with 5 vertices. (2)

(b) For the graph I in **Q3(b)**, find a spanning tree with root f,

(i) using *depth-first* search; (1)

(ii) using *breadth-first* search. (1)

(c) Using alphabetical order, form a binary search tree for the words:
Lily, Daisy, Rose, Violet, Orchid, Tulip, Lotus. (2)

Q5. **(a)** For the Boolean function $f(x, y, z) = (x + y\overline{z})(\overline{x} + y)$, find

(i) the complete sum-of-products expansion (CSP); (2)

(ii) the complete product-of-sums expansion (CPS). (2)

(b) Let $g(x, y, z) = xyz + x\overline{y}\,\overline{z} + x\overline{y}z + \overline{x}\,y\overline{z} + \overline{x}\,\overline{y}z$ be a Boolean function.
(i) Build the Karnaugh map of g. (1)

(ii) Simplify g (i.e., write it in MSP form). (2)