College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Final Exam
Academic Year 1445 Hijri- SecondSemester

Exam Information معلومات الامتحان			
Course name	Discrete Mathematics الرياضيات المحددة		اسم المقرر
Course Code	151 ريض		رمز المقرر
Exam Date	2024-06-03	1445-11-26	تاريخ الامتحان
Exam Time	08: 00 AM		وقت الامتحان
Exam Duration	3 hours	ثلاث ساعات	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 9 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
- Calculators are not allowed.

تعليمات عامة:

- عدد صفحات الامتحان 9 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
- الآلة الحاسبة ممنوعة.

هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	1.1	2a(ii,iii)		
2	1.2	2b(iii), 4b		
3	2.1	1, 3(e,f), 4a		
4	2.2	2a(i), 2b(i,ii)		
5	2.3	3(a to d)		
6	2.4	4c		
7				
8				

EXAM COVER PAGE

Q	1	2ai,ii	2aiii	2bi,ii	2biii	3a-d	3e,f	4a	4b	4c	Total
Grade											

Q1. (a) Without using truth tables, show that $\neg(p \wedge q) \wedge (p \rightarrow \neg r)$ is logically equivalent to $(q \vee r) \rightarrow \neg p$. (3 points)

(b) For any integers x and y, prove by contraposition that: if $x^2(y+3)$ is even then x is even or y is odd. (2 points)

(c) Use induction to prove that $9 + 13 + 17 + \ldots + (4n + 5) = n(2n + 7)$ for all $n \geq 1$. (4 points)

Q2. **(a)** Let A be the set of **even** integers, and let E be the relation on A defined by aEb if and only if 4 divides $a + b$.

(i) Show that E is an equivalence relation. (3 points)

(ii) Find [0]. (1 point)

(iii) Is -6 related to 12? (Justify your answer.) (1 point)

(b) Let $P = \{(1,1),(1,3),(1,4),(2,2),(3,3),(3,4),(4,4)\}$ be a relation on $B = \{1,2,3,4\}$.

(i) Show that P is a partial ordering. (3 points)

(ii) Is P a total ordering? (Justify your answer.) (1 point)

(iii) Represent P with a Hasse diagram. (1 point)

Q3. (a) Let G be a graph with 14 edges and degree sequence $a, a, 5, 5, 5, 5$.
(i) Find a. (1 point)

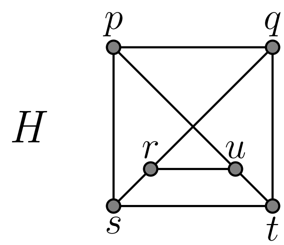
(ii) Can G be a complete graph? (Justify your answer.) (1 point)

(b) Let M be a (undirected) graph represented with the following adjacency matrix.

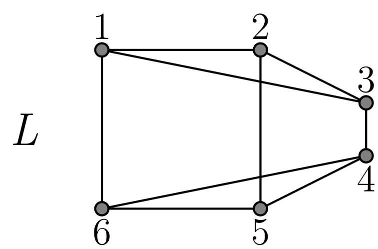
$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Draw M, and show that it is not a tree. (2 points)

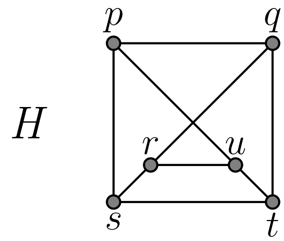
(c) Determine whether the following graph H is bipartite, and if so, then find a bipartite representation. (2 points)



(d) Determine whether the graph H in **(c)** is isomorphic the graph L below. (Justify your answer.) (2 points)



(e) For the graph H below, find a spanning tree with root r,



(i) using *depth-first* search; (1 point)

(ii) using *breadth-first* search. (1 ponit)

(f) Using alphabetical order, form a binary search tree for the words:
Saw, Hammer, Screwdriver, Ax, Pliers, Wrench, Drill. (2 points)

Q4. (a) Without using tables, prove the following Boolean identity:

$$\overline{\overline{(\overline{x}+y)}+z} = \overline{x}\;\overline{z} + y\overline{z}. \text{ (2 points)}$$

(b) (i) Find the complete sum-of-products expansion (CSP) for $f(x, y, z) = (x\overline{z} + y)(x + yz)$. (2 points)

(ii) Find the complete product-of-sums expansion (CPS) for $g(x, y, z) = \overline{x}z + y$. (2 points)

(c) Let $h(x, y, z) = xyz + xy\overline{z} + x\overline{y}z + \overline{x}yz + \overline{x}\,\overline{y}\,\overline{z} + \overline{x}\,\overline{y}z$ be a Boolean function.
(i) Build the Karnaugh map of h. (1 point)

(ii) Simplify h (i.e., write it in MSP form). (2 points)

Good Luck :)