College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Final Exam
Academic Year 1445 Hijri- First Semester

Exam Information معلومات الامتحان		
Course name	**Discrete Mathematics**	**اسم المقرر**
Course Code	**Math 151**	**رمز المقرر**
Exam Date	**2023-12-25**	**1445-06-12**	**تاريخ الامتحان**
Exam Time	**08: 00 AM**	**وقت الامتحان**
Exam Duration	3 hours	**ثلاث ساعات**	**مدة الامتحان**
Classroom No.		**رقم قاعة الاختبار**
Instructor Name		**اسم استاذ المقرر**

Student Information معلومات الطالب		
Student's Name		**اسم الطالب**
ID number		**الرقم الجامعي**
Section No.		**رقم الشعبة**
Serial Number		**الرقم التسلسلي**

General Instructions:

- Your Exam consists of 9 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
- Calculators are not allowed.

تعليمات عامة:

- عدد صفحات الامتحان 9 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
- يمنع استخدام الآلة الحاسبة.

هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	1.1	2a(ii)		
2	1.2	5b		
3	2.1	1, 3(e,f), 4a		
4	2.2	2a(i), 2b		
5	2.3	3(a to d)		
6	2.4	4c		
7				
8				

EXAM COVER PAGE

Question	Grade
1	
2a(i)	
2a(ii)	
2b	
3 (a,b,c,d)	
3 (e,f)	
4a	
4b	
4c	
Total	

Q1. (a) Without using truth tables show that $(\neg p \to q) \to p \equiv q \to p$. (3 points)

(b) Use induction to show that $n^3 + 5n$ is divisible by 3 for all $n \geq 0$. (4 points)

(c) Suppose a and b are integers. Use contraposition to prove that if $a^2 - b^2 = 33$, then a is even or b is even. (3 points)

Q2. (a) Define a relation E on $\mathbb{Z} - \{0\}$ as mEn if and only if $3mn > 0$.
(i) Show that E is an equivalence relation. (3 points)

(ii) Find $[1]$ and $[-1]$. (2 points)

(b) Define a relation R on $\mathbb{Z}$ as xRy if and only if $x^2 \geq y^2$.

(i) Determine whether R is reflexive, symmetric, antisymmetric or transitive. (4 points)

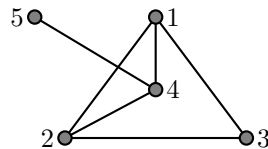(ii) Is R a partial ordering? (Justify your answer.) (1 point)

Q3. **(a)** Show that an undirected graph with degree sequence $4, 3, 2, 2, 1, x$ cannot be connected if it has exactly 6 edges. (2 points)

(b) Find the number of vertices n of the complete graph K_n, that has 36 edges. (2 points)

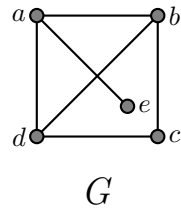**(c)** Determine if the graphs G and H below are isomorphic. (Justify your answer.) (2 points)
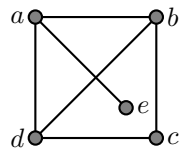
G H

(d) Is the graph G below bipartite? (Justify your answer.) (1 point)

G

(e) For the graph G in **(d)**, find a spanning tree with root a,
(i) using *depth-first* search; (1 point)

(ii) using *breadth-first* search. (1 point)

(f) Using alphabetical order, form a binary search tree for the words: *Mother, Father, Son, Daughter, Aunt, Uncle, Cousin.* (2 points)

Q4. (a) Prove the following Boolean identity:

$$\overline{x} + xy = \overline{x\overline{y}}. \ (2 \text{ points})$$

(b) Let $f(x, y, z) = x(\overline{y} + x\overline{z})$ be a Boolean function.

(i) Find the complete sum-of-products expansion (CSP) of f. (2 points)

(ii) Find the complete product-of-sums expansion (CPS) of f. (2 points)

(c) Let $g(x, y, z) = xyz + x\overline{y}\,\overline{z} + \overline{x}y\overline{z} + \overline{x}\,\overline{y}\,\overline{z} + \overline{x}\,\overline{y}z$ be a Boolean function.

(i) Build the Karnaugh map of g. (1 point)

(ii) Simplify g (i.e., write it in MSP form). (2 points)

Good Luck :)