| Question | I | II | III | IV | Total |
|---|---|---|---|---|---|
| Mark | | | | | |

**Question I:** (0.5×8=4points)

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Answer | b | a | c | a | b | b | c | a |

Choose the correct answer, then fill in the table above:

(1) If a relation $R$ is defined on $\mathbb{Z}$, by $aRb \leftrightarrow a + b \text{ is prime}$, then the following pair belongs to $R$

(a) (1,3).
(b) (3,4).
(c) (2,7).
(d) None of the previous.

(2) If $R = \{(a,b)|a > b^2\}$ is a relation defined on the set on real number, then $\bar{R} =$

(a) $\{(a,b\}|a \leq b^2\}$.
(b) $\{(a,b\}|a > b^2\}$.
(c) $\{(a,b\}|a^2 \leq b\}$.
(d) None of the previous.

(3) In the set of positive integers with division relation $(\mathbb{Z}^+, |)$, the following pairs are comparable:
(a) 2, 31.
(b) 21, 14.
(c) 4, 12.
(d) None of the previous.

(4) If $R$ and $S$ are relations on a set $A$ represented by the matrices

$$M_R = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \text{ then } M_{S \circ R} =$$

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$.

(b) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.

(c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

(d) None of the previous.

(5) Let $A = \{a, b, c\}$ and the relation $R$ on $A$ be represented by the matrix

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Then the relation $R$ is

(a) Reflexive.

(b) Antisymmetric.

(c) Symmetric.

(d) None of the previous.

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(6) Let $A = \{1, 2, 3, 4\}$. The equivalence relation produced by the partition $A_1 = \{1, 4\}$, $A_2 = \{2\}$, $A_3 = \{3\}$ is given by:

(a) $\{(1,1), (1,4), (2,2), (3,3), (4,4)\}$.

(b) $\{(1,1), (1,4), (4,1), (2,2), (3,3), (4,4)\}$.

(c) $\{(1,1), (1,4), (4,1), (2,2), (2,3), (3,2), (3,3), (4,4)\}$.

(d) None of the previous.

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(7) Let $G = (V, E)$ be an $r - regular$ graph with $|V| = |E|$. Then the value of $r$ is

(a) 16.

(b) 4.

(c) 2.

(d) None of the previous.

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(8) Let $H$ be a graph with 8 edges and degree sequence $1, 3, x, x$, then $x =$

(a) 6.

(b) 2.

(c) 1.

(d) None of the previous.

**Question II**: (2+4=6 points)

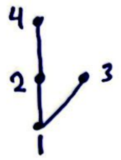
A. For the partial ordered set on $A = \{1, 2, 3, 4\}$, given by:

$$S = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\},$$

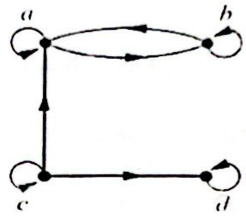
draw the **Hasse diagram** representing the relation S. Is the poset **a totally ordered set**? Justify your answer.

No, because 2 and 3 are incomparable

$(2,3) \notin S$ and $(3,2) \notin S$




B. Let $R$ be a relation on $A = \{a, b, c, d\}$, represented by the directed graph (digraph) below:




Then answer the following:

(i) List all ordered pairs of $R$.

$R = \{(a,a), (b,b), (c,c), (d,d), (a,b), (b,a), (c,a), (c,d)\}$

(ii) Is the relation $R$ reflexive? Justify your answer.

Yes, because $(x,x) \in R \ \forall x \in A$ (or $(a,a) \in R$, $(b,b) \in R$, $(c,c) \in R$, $(d,d) \in R$)

or " there is a loop at every element of A.

(iii) Is the relation $R$ symemetric? Justify your answer.

No, because $(c,a) \in R$ but $(a,c) \notin R$

(iv) Is the relation $R$ antisymmetric? Justify your answer.

No, because $(a,b) \in R$ and $(b,a) \in R$

but $a \neq b$

**Question III**: (4.5+1.5+1+1=8 points)

Let $R$ be a relation defined on the set of integers $\mathbb{Z}$ by $R = \{(a,b) : a - b \text{ even}\}$. Then answer the following:

(i) Prove that $R$ is an equivalence relation on $\mathbb{Z}$.

① $\forall a \in \mathbb{Z}$, $a - a = 0 = 2(0)$ is even $\Rightarrow (a,a) \in R \ \forall a \in \mathbb{Z}$

$\therefore R$ reflexive

② $\forall a,b \in \mathbb{Z}$, if $(a,b) \in R \Rightarrow a - b = 2k$ for some $k \in \mathbb{Z}$

$\Rightarrow b - a = 2(-k) = \underline{\underline{2m}}$, where $m := -k \in \mathbb{Z}$

$b - a = 2m$ . is even

$\therefore (b,a) \in R \quad \therefore R$ symmetric

③ $\forall a,b,c \in \mathbb{Z}$, If $(a,b) \in R$ and $(b,c) \in R \Rightarrow a - b = 2k_1 \wedge b - c = 2k_2$ for some $k_1, k_2 \in \mathbb{Z}$

$\Rightarrow (a - \cancel{b}) + (\cancel{b} - c) = 2k_1 + 2k_2 = 2(k_1 + k_2)$

$\Rightarrow a - c = 2m$, where $m = k_1 + k_2 \in \mathbb{Z}$

$a\ c$ is even

$\therefore (a,c) \in R \quad \therefore R$ transitive.

(ii) Find the equivalence classes [0] and [1].

$[0] = \{a \mid (a,0) \in R\} = \{a \mid a - 0 \text{ even}\} = \{a \mid a - 0 = 2k, \text{ for some } k \in \mathbb{Z}\}$

$= \{\dots, -4, -2, 0, 2, 4, \dots\} = 2\mathbb{Z}$.

$[1] = \{a \mid (a,1) \in R\} = \{a \mid a - 1 = 2k, k \in \mathbb{Z}\} = \{2k+1, \text{ for some } k \in \mathbb{Z}\} = \{\dots, -3, -1, 1, 3, 5, \dots\}$

(iii) Use (ii) to find a partition of $\mathbb{Z}$.

$[0]$ and $[1]$ form a partition of $\mathbb{Z}$.

1. $[0] \neq \phi$, $[1] \neq \phi$
2. $[0] \cap [1] = \phi$
3. $[0] \cup [1] = \mathbb{Z}$.

(iv) Is the relation $R$ a partial ordered set on $\mathbb{Z}$? Justify your answer.

No, because $R$ not antisymmetric

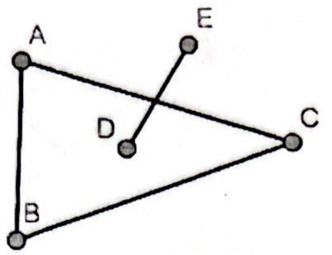
$(4 - 2 = 2 \text{ even})$

since $\underset{(2-4 = -2 \text{ even})}{(2,4) \in R}$ and $(4,2) \in R$

but $2 \neq 4$.

**Question IV:** ( 2.5+3+1.5=7points)

A. Answer the following questions about the graph $G$:




$G$

(i) Is the graph $G$ connected? Justify your answer.

No, because there is no Path between the vertex (A) and the Vertex (E)

(ii) How many connected components are in the graph $G$?

There are 2 connected components.

(iii) Find a simple circuit. What is its length?

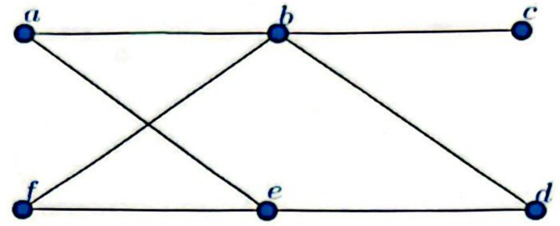
A, B, C, A

its length = 3

(iv) What is the degree of $C$.

degree C = 2

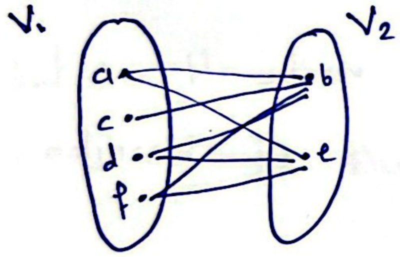
B. Answer the following questions about the graph $J$ below




$J$

(i) Is the graph $J$ bipartite? Justify your answer.

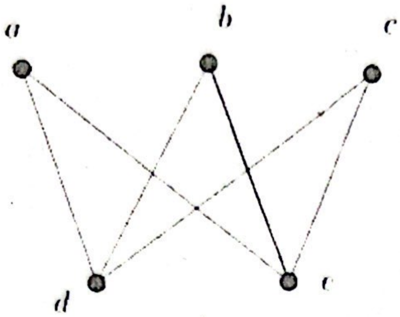
yes, because its vertex set is the union of two disjoint sets, $V_1 = \{a, c, d, f\}$ and $V_2 = \{b, e\}$, and each edge connects a vertex in $V_1$ to a vertex in $V_2$.



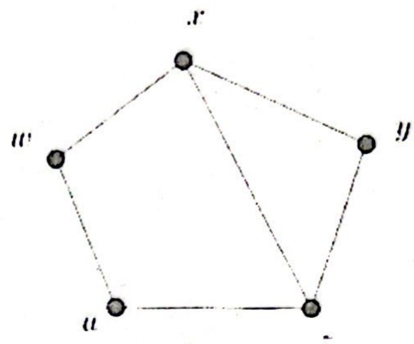

(ii) Represent the graph $J$ in an **adjacency matrix**.

$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

C. Are the graphs $G$ and $H$ below isomorphic? Justify your answer.




$G$




$H$

No, $H \ncong G$

because each vertex in G of degree 3 is adjacent to another vertex of degree 2

but in H there is a vertex ($x$) of degree 3 and is adjacent to the vertex Z of degree 3.

Good Luck ☺