

Quiz (2) ⁽²⁾
الاجابة النموذجية



لا يكتب في
هذا الهامش

Q1 Prove that if x is an odd integer, then $9x+5$ is even

Method (1) direct proof $P \rightarrow Q$

Assume x is an odd integer

$$\text{So, } x = 2m+1, m \in \mathbb{Z}$$

$$\text{Now, } 9x+5 = 9(2m+1)+5$$

$$= 18x+9+5 = 18x+14$$

$$= 2(9x+7) = 2r, r=9x+7 \in \mathbb{Z}$$

i.e. $9x+5$ is even.



Q2 Prove that, if $n \in \mathbb{Z}$

$(n+1)^2 - 1$ is even $\Leftrightarrow n$ is even

(i) $(\Rightarrow)$ Assume $(n+1)^2 - 1$ is even $\Rightarrow$
to show that n is even.

"Proof by contrapositive" ($P \rightarrow Q \equiv \neg Q \rightarrow \neg P$)

"If n is odd $\Rightarrow n = 2k+1 \Rightarrow$

$$(n+1)^2 - 1 = (2k+2)^2 - 1 = 4k^2 + 8k + 3$$

$$= 2(k^2 + 4k + 1) + 1 = 2r + 1$$

(such that $r = k^2 + 4k + 1 \in \mathbb{Z}$)

So, $(n+1)^2 - 1$ is odd

(ii) $(\Leftarrow)$ Assume n is even $\Rightarrow$ show that

$(n+1)^2 - 1$ is even also

$$\text{Now, } n = 2k \Rightarrow (n+1)^2 - 1 = (2k+1)^2 - 1$$

$$= 4k^2 + 4k = 2(k^2 + 2k)$$

$$= 2t, t = k^2 + 2k \in \mathbb{Z}$$

$\Rightarrow (n+1)^2 - 1$ is even.

~~Quiz~~

Q₁) Prove that if x is an ~~old~~ odd integer, then $9x+5$ is even

Q₂) Let $n \in \mathbb{Z}$, Prove that:
 $(n+1)^2 - 1$ is even if and only if n is even