First semester	First exam	King Saud University
(1^{st} Oct. 2025)-(9/4/1447)	131 Math	College of Science
Without calculators	Time: 90 minutes	Mathematics Department

Name: **University Number:**

Q_1: Using symbols, definitions and theorems that you have studied, fill in the blanks, where U is the universal set: (12 marks)

1- $\sim(\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x+1<y) \equiv \cdots\cdots$
2- $\overline{\mathrm{A}} = \{\cdots \mid \cdots\cdots\}$ and $\mathrm{A} \cup \mathrm{B} = \{\cdots \mid \cdots\cdots\}$
3- A compound statement S is called a **tautology** if ....
4- $\sim(A \vee (\sim B)) \equiv \cdots\cdots$
5- If A={1,2} and B={3}, then A×B=...... and the power set p(A)=......
6- If A and B are disjoint sets such that |A∪B|=5 and |A−B|=2, then |B|=..., |A∩B|=..., |A×B|=... and |p(A)|=....

Q_2: Answer the following questions:

1- Use the truth table to show that $A \wedge (B \wedge C) \equiv (A \wedge B) \wedge (A \wedge C)$ (4 marks)
2- Show that if $3x^4+1$ is even then $5x+2$ is odd. (4 marks)
3- Show that $P = \{\mathbb{Z}^+, \mathbb{Z}^-, \{0\}\}$ is a partition of the set of integer numbers. (3 marks)
4- Evaluate the proposed proof of the following result: "If x and y are positive odd numbers, then $x+y$ is even".
   Proof: take the odd numbers x=1 and y=3. Observe that $x+y$=1+3=4 which is an even number. (2 marks)

<u>Answers</u>

Q_1:

1- $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x+1 \geq y$
2- $\overline{\mathrm{A}} = \{x | x \in U \text{ and } x \notin A\}$
$\mathrm{A} \cup \mathrm{B} = \{x | x \in A \text{ or } x \in B\}$
3-
if it is true for all possible combinations of truth values of the component statements that comprise *S*.
4- $\sim(A \vee (\sim B)) \equiv (\sim \mathrm{A}) \wedge \mathrm{B}$
5- A×B={(1,3),(2,3)} and p(A)={∅,{1},{2},A}
6- |B|=3, |A∩B|=0, |A×B|=6, $|p(A)|=2^2=4$

Q_2:

1-

A	B	C	$A \wedge B$	$A \wedge C$	$B \wedge C$	$A \wedge (B \wedge C)$	$(A \wedge B) \wedge (A \wedge C)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	F	F	F	F
F	T	T	F	F	T	F	F
F	T	F	F	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

From the 7^{th} and 8^{th} columns, it is clear that they are logically equivalent.

2- The proof consists of two steps:

Step 1: If $3x^4+1$ is even, then x is odd using the proof by contrapositive. So, suppose that x is even. That implies that $x=2k$ for some integer k. Then we have that:

$$3x^4+1=3(2k)^4+1=3(2)^4k^4+1=48\,k^4+1=2(24\,k^4)+1$$

Since $24\,k^4$ is an integer, $3x^4+1$ is odd. So, we proved that if x is even, then $3x^4+1$ is odd. The contrapositive of this statement is: if $3x^4+1$ is even, then x is odd.

Step 2: If x is odd, then $5x+2$ is odd. We will use the direct proof. Since x is odd, we have that $x=2k+1$ for some integer k. So,

$5x+2=5(2k+1)+2=10k+7=10k+6+1=2(5k+3)+1$

Since $5k+3$ is an integer, $5x+2$ is odd.

3- Each element in P is nonempty, the intersection of every two different elements in P is empty and the union of the three elements in P is $\mathbb{Z}$.

4- The proof is incorrect since it took specific numbers to prove the result while it should be taken arbitrary odd numbers (not necessary x is different from y).