حل نموذجي

College of Science.
Department of Mathematics

جامعة الملك سعود
King Saud University
كلية العلوم
قسم الرياضيات

Second Midterm Exam
Academic Year 1444-1445 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	Integral Calculus		اسم المقرر
Course Code	Math111 ريض 111		رمز المقرر
Exam Date	2023-11-01	1445-04-17	تاريخ الامتحان
Exam Time	03: 00 PM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name	حنان العوهلي		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 6 صفحة. (باستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	CLO 1.1	QI(2,3,4,5,6)-QII(B)	6.5	
2	CLO 1.2	QI(1)	1	
3	CLO 2.1	QIV	10	
4	CLO 2.2	QII(A)-QIII(B)	4.5	
5	CLO 2.4	QIII(A)	3	25
6				
7				
8				

EXAM COVER PAGE

Question	Mark
Question I	
Question II	
Question III	
Question IV	
Total	

Question I: (6 points)

Question Number	1	2	3	4	5	6
Answer	c	b	c	a	c	a

Choose the correct answer, then fill in the table above:

(1) If f is smooth and $f(x) \geq 0$ on $[a, b]$, then the area S of the surface generated by revolving the graph of f about the $x-axis$ is

(a) $2\pi \int_a^b f'(x)\sqrt{1+[f'(x)]^2}\, dx$

(b) $2\pi \int_a^b \sqrt{1+[f'(x)]^2}\, dx$

(c) $2\pi \int_a^b f(x)\sqrt{1+[f'(x)]^2}\, dx$

(d) None of the previous

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(2) If $e^{\ln 2x} = 4$, then x equals

(a) 1

(b) 2

(c) 4

(d) None of the previous

Question II: (3+1.5 points)

A. Use logarithmic differentiation to find $\frac{dy}{dx}$, for the function

$$y = \frac{(x-2)\,(x^2+3)^5}{\sqrt{x+1}}$$

$$\ln y = \ln(x-2) + \ln(x^2+3)^5 - \ln(x+1)^{1/2}$$

$$= \ln(x-2) + 5\ln(x^2+3) - \tfrac{1}{2}\ln(x+1)$$

$$\frac{1}{y}y' = \frac{1}{x-2} + 5\left(\frac{2x}{x^2+3}\right) - \frac{1}{2}\,\frac{1}{x+1}$$

$$y' = y\left[\frac{1}{x-2} + \frac{10x}{x^2+3} - \frac{1}{2(x+1)}\right] = \frac{(x-2)(x^2+3)^5}{\sqrt{x+1}}\left[\frac{1}{x-2} + \frac{10x}{x^2+3} - \frac{1}{2x+2}\right]$$

B. If $f(x) = sec^{-1}(\sinh x)$, Find $f'(x)$

$$f'(x) = \frac{1}{\sinh x\sqrt{\sinh^2 x - 1}} \cdot \cosh x .$$

Question III: (3+1.5 points)

A. Find the arc length of $f(x) = \cosh x,\ 0 \le x \le 1$.

$$L = \int_a^b \sqrt{1+f'(x)^2}\ dx = \int_0^1 \sqrt{1+\sinh^2 x}\ dx$$

$$= \int_0^1 \sqrt{\cosh^2 x} \cdot dx = \int_0^1 \cosh x\ dx$$

$$= \sinh x\Big]_0^1 = \sinh 1 - \sinh 0 .$$

B. Solve for x

$$\ln|x^2 - 1| - \ln|x - 1| = 0$$

$$\ln\left|\frac{x^2-1}{x-1}\right| = 0$$

$$\ln\left|\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}\right| = 0$$

$$\ln(x+1) = 0$$

$$x+1 = 1$$

$$x = 0$$

Question IV: (10 points)

Evaluate the following integrals:

(1) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

$u = \sqrt{x}$ $\quad du = \frac{1}{2\sqrt{x}}\, dx$

$2\, du = \frac{1}{\sqrt{x}}\, dx$

$= 2\int e^{u}\, du$

$= 2\, e^{u} + C = 2\, e^{\sqrt{x}} + C$.

(2) $\int \frac{x}{9+x^4}\, dx$

$= \int \frac{x}{9+(x^2)^2}\, dx$

$u = x^2 \,,\quad du = 2x\, dx$

$\frac{1}{2}\, du = x\, dx$

$= \frac{1}{2}\int \frac{1}{9+u^2}\, du =$

$\frac{1}{2}\frac{1}{3}\tan^{-1}\frac{u}{3} + C = \frac{1}{6}\tan^{-1}\frac{x^2}{3} + C$

(3) $\int x^2 \ln|x|\, dx$

$u = \ln|x|$ $\quad dv = x^2\, dx$

$du = \frac{1}{x}\, dx$ $\quad v = \frac{x^3}{3}$

$I = uv - \int v\, du = \frac{x^3}{3}\ln|x| - \frac{1}{3}\int \frac{1}{\cancel{x}} \cdot \underbrace{x^3}_{x^2}\, dx$

$= \frac{x^3}{3}\ln|x| - \frac{1}{3}\,\frac{x^3}{3} + C$

(4) $\int tan^2x\, sec^4x\ dx$

$$\int \tan^2 x \sec^2 x \cdot \sec^2 x\, d$$

$$= \int \tan^2 x\,[1+\tan^2 x]\,\sec^2 x\, dx$$

$u = \tan x$

$du = \sec^2 x\, dx$

$$= \int u^2[1+u^2]\, du$$

$$= \int (u^2 + u^4)\, du = \frac{u^3}{3} + \frac{u^5}{5} + C$$

$$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$$

(5) $\int x \cos x\ dx$

$u = x \qquad dv = \cos x\, dx$

$du = dx \qquad v = \sin x$

$$I = x\sin x - \int \sin x\, dx$$

$$= x\sin x + \cos x + C$$

Good Luck☺