Model
Answer

College of Science.
Department of Mathematics

جامعة الملك سعود
King Saud University

كلية العلوم
قسم الرياضيات

First Midterm Exam
Academic Year 1443-1444 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	Integral Calculus		اسم المقرر
Course Code	Math111 ريض 111		رمز المقرر
Exam Date	2023-10-04	1445-03-19	تاريخ الامتحان
Exam Time	03: 00 PM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name	حنان العوهلي		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 6 صفحة. (باستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	CLO 2.1	QV	4	
2	CLO 2.2	QI+QII+QIV	5+5+3	
3	CLO 2.4	QIII	8	
4				
5				
6				
7				

EXAM COVER PAGE

Question V: (4 points)

Evaluate the following integrals:

(i) $\int_1^3 |x-2|\, dx$

$$|x-2| = \begin{cases} x-2 & x \geq 2 \\ 2-x & x < 2 \end{cases}$$

$$= \int_1^2 2-x \quad dx + \int_2^3 x-2 \quad dx$$

$$= 2x - \frac{x^2}{2}\Big]_1^2 + \frac{x^2}{2} - 2x\Big]_2^3$$

$$= 4 - 2 - (2 - 1/2) + \frac{9}{2} - 6 - (2 - 4)$$

$$= \underbrace{8 - 4 - 2}\, + \underbrace{1/2 + 9/2}_{5} - 6 = -4 + 5 = 1$$

(ii) $\int \frac{(\sqrt{x}+3)^4}{\sqrt{x}}\, dx$

$$u = \sqrt{x} + 3 \qquad du = \frac{1}{2\sqrt{x}}\, dx \qquad 2du = \frac{1}{\sqrt{x}}\, dx$$

$$= 2\int u^4\, du$$

$$= 2\,\frac{u^5}{5} + c = \frac{2}{5}(\sqrt{x}+3)^4 + C$$

Good Luck☺

Question	Mark
Question I	
Question II	
Question III	
Question IV	
Question V	
Total	

Question I: (5 points)

Question Number	1	2	3	4	5
Answer	a	c	b	c	c

A. Choose the correct answer, then fill in the table above:

(1) If $f(x) = 4x^3 + \cos x$, then the most general antiderivative of f is

(a) $x^4 + \sin x + C$ (b) $12x^4 + \sin x + C$
(c) $12x^2 + \sec x + C$ (d) None of the previous

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(2) If $\int_1^3 f(x)dx = 5$, $\int_3^1 g(x)dx = 2$, then $\int_1^3[3f(x) + g(x)]dx =$

(a)3 (b) 17 (c) 13 (d) None of the previous

$3\int_1^3 f + \int_1^3 g = 3(5) - 2$
$= 13$

(3) $\sum_{k=1}^{5}(k-\alpha)=7$, then the value of α is

(a) $\frac{22}{5}$ (b) $\frac{8}{5}$ (c) $\frac{12}{5}$ (d) None of the previous

$\frac{5(6)}{2} - 5\alpha = 7$

$15 - 5\alpha = 7$ $5\alpha = 8$ $\alpha = 8/5$

2

1x

(4) If $F(x)=\int_1^{3x^2+1}\tan t\,dt$, then $F'(x)=$

(a) $\tan(3x^2+1)$ (b) $6x\sec^2(3x^2+1)$

(c) $6x\tan(3x^2+1)$ (d) None of the previous

(5) $\int D_x[x^5\sin^3 x]dx =$

(a) $\frac{1}{6}x^6\cos^3 x + C$ (b) $x^5\sin^3 x$

(c) $x^5\sin^3 x + C$ (d) None of the previous

Question II: (2+3 points)

$y-5=x^2$

$x^2+5=x+7 \Rightarrow x=2$, $x=-1$

A. Without solving the integral prove that

$$\int_0^2 (x^2+5)dx \le \int_0^2 (x+7)dx.$$

$x^2+5 \le x+7$?

$\Leftrightarrow x^2-x-2 \le 0$??

$x^2-x-2 = (x-2)(x+1)$

$x=2 \qquad x=-1$ 0.5

$\therefore$

		-1		2	
x^2-x-2	$+$		$--$		$+$

0.5

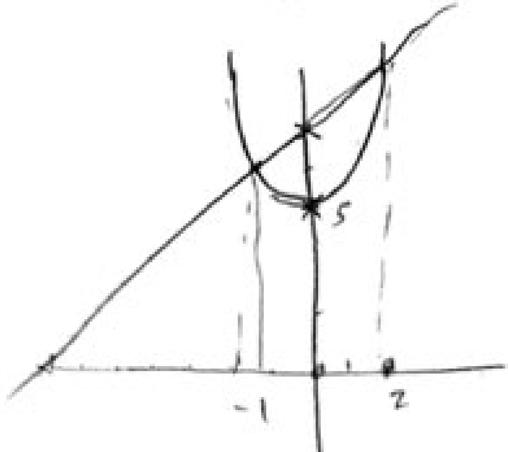
$\Rightarrow \forall -1 \le x \le 2$, $x^2-x-2 \le 0$

$\Rightarrow \forall\ 0 \le x \le 2$, $x^2-x-2 \le 0$

0.5

$x^2+5 \le x+7 \quad \forall\ 0 \le x \le 2$

$\Rightarrow \int_0^2 (x^2+5)dx \le \int_0^2 (x+7)\,dx$



B. Find the value of z that satisfies the conclusion of the Integral Mean Value Theorem for $f(x)=x^3$ on [2,4].

$\int_a^b f(x)\,dx = f(z)(b-a)$ 0.5

$\int_2^4 x^3\,dx = \frac{x^4}{4}\Big]_2^4 = \frac{4^4}{4} - \frac{2^4}{4} = 64-4=60$ 1

0.5 0.5

$z^3(4-2)=60 \qquad z^3=30$

$\boxed{z=\sqrt[3]{30} \in (2,4)}$

0.5

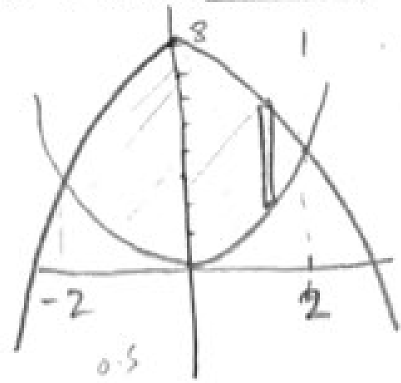
Question III: (4+~~4~~ 3 points) $\quad y-8=-x^2$

A. Sketch the region R bounded by the graphs of the functions $y = x^2$, $\quad y = 8 - x^2$ and then **find its area.**

نقاط التقاطع $\quad$ 0.5

$x^2 = 8 - x^2 \Rightarrow 2x^2 = 8$

$x^2 = 4 \qquad x = \pm 2$



$$A = \int_{-2}^{2} (8-x^2) - x^2 \, dx \qquad 0.5 \quad 1$$

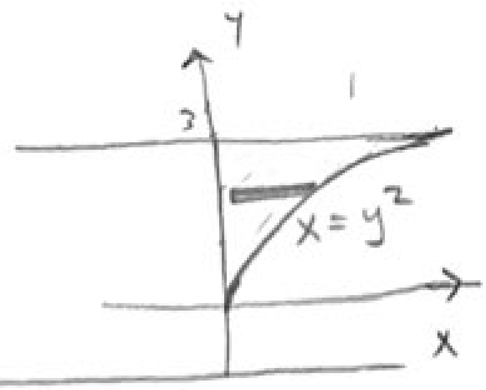
$$= \int_{-2}^{2} (8-2x^2)\,dx = 8x - \frac{2x^3}{3}\Big]_{-2}^{2} = 8(2) - \frac{2}{3}(8) - \left(-16 - \frac{2}{3}(-8)\right)$$

0.5 $\quad$ 0.5

$$= 16 - \frac{16}{3} + 16 - \frac{16}{3} = 32 - \frac{32}{3} \approx 21.33 \quad \frac{64}{3}$$

B. Let R be the region bounded by the graphs of $y = \sqrt{x}$, $\quad y = 3$ and y-axis. **Sketch the region R** and **set up the integral** for the volume of the solid resulting by revolving R about

(i) The x-axis. $\qquad$ (ii) The y-axis. $\quad \sqrt{x} = 3 \Rightarrow x = 9$



1 (i) cylinder

$$V = 2\pi \int_0^3 y \cdot y^2 \, dy$$

$$= 2\pi \int_0^3 y^3 \, dy$$

1 (ii) disk

$$V = \pi \int_0^3 (y^2)^2 \, dy$$

$$= \pi \int_0^3 y^4 \, dy$$

$\boxed{\text{or}}$

(i)
$$V = \pi \int_0^9 3^2 - (\sqrt{x})^2 \, dx$$

$$= \pi \int_0^9 (9-x)\,dx$$

(ii) $$V = 2\pi \int_0^9 x(3-\sqrt{x})\,dx$$

Question IV: (3 points)

Find the area under the curve $f(x) = 3x^2 + 1$ on $[0,4]$, by taking the limit of the Riemann sum and the right-handed endpoints.

(1) $\Delta x = \frac{b-a}{n} = \frac{4-0}{n} = \frac{4}{n}$

(2) $w_k = x_k = a + k\,\Delta x = 0 + k\left(\frac{4}{n}\right) = \frac{4k}{n}$

(3) $f(w_k) = f\left(\frac{4k}{n}\right) = 3\left(\frac{4k}{n}\right)^2 + 1 = 3\,\frac{16k^2}{n^2} + 1 = \frac{48k^2}{n^2} + 1$

(4) $R_p = \sum_{k=1}^{n} f(w_k)\,\Delta x = \sum_{k=1}^{n}\left(\frac{48k^2}{n^2} + 1\right)\frac{4}{n}$

$= \frac{4}{n}\sum_{k=1}^{n}\left(\frac{48k^2}{n^2} + 1\right) = \frac{4}{n}\left[\frac{48}{n^2}\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1\right]$

$= \frac{4}{n}\left[\frac{48}{n^2}\,\frac{n(n+1)(2n+1)}{6} + n\right] = \frac{4}{n}\left[8\left(\frac{2n^2+3n+1}{n}\right) + n\right]$

$= 32\,\frac{(2n^2+3n+1)}{n^2} + 4$

(5) $A = \lim_{n\to\infty} R_p = \lim_{n\to\infty}\left(32\,\frac{(2n^2+3n+1)}{n^2} + 4\right)$

$= 32(2) + 4 = 68$