	Q1	Q2	Q3	Total

Question 1: (2+2+2)

1. Find the value of c that satisfies the Mean Value Theorem for definite integrals for the function $f(x) = (x+1)^2$ on the interval $[-1,2]$.

$\int_a^b f(x)\,dx = f(c)\,(b-a) \Longleftrightarrow \int_{-1}^{2} (x+1)^2\,dx = 3(c+1)^2$

$\boxed{1}$ $\int_{-1}^{2} (x+1)^2\,dx = \frac{(x+1)^3}{3}\Big|_{-1}^{2} = \frac{27}{3} - 0 = 9$

So $3(c+1)^2 = 9 \Rightarrow (c+1)^2 = 3 \Rightarrow c+1 = \pm\sqrt{3}$

only $+\sqrt{3}-1 \in [-1,2]$, so $\boxed{c = \sqrt{3} - 1}$

2. If $F(x) = \int_{\sec x}^{\log_2 x} \frac{1}{\sqrt{1+t}}\,dt$, find $F'(x)$. $= f(h(x)).h' - f(g(x))\,g'(x)$

$h(x) = \log_2 x = \frac{\ln x}{\ln 2} \to h' = \frac{1}{x\ln 2}$

$g(x) = \sec x = \sec x \Rightarrow g'(x) = \sec x \tan x$

so $f'(x) = \frac{1}{x\ln 2\sqrt{1+\log_2 x}} - \frac{\sec x \tan x}{\sqrt{1+\sec x}}$

3. If $y(x) = \ln\left|\frac{(x^2+1)^{\frac{2}{3}}\cos^7(5x)}{\sqrt[3]{x^4+1}}\right|$, find $\frac{dy}{dx}$.

rewrite $y(x)$ by uesing proprties of ln function

$y = \ln(x^2+1)^{\frac{2}{3}} + \ln\cos^7(5x) - \ln\sqrt[3]{x^4+1}$

$y = \frac{3}{2}\ln(x^2+1) + 7\ln\cos(5x) - \frac{1}{3}\ln(x^4+1)$

$y' = \frac{3}{2}\,\frac{2x}{x^2+1} + \frac{\cancel{35}\sin 5x}{\cos(5x)} - \frac{1}{3}\,\frac{4x}{x^4+1}$

...n 2: (2+2+3+3+3+3)

...uate the following integrals:

1. $\int \frac{1}{x+x\ln x}\,dx$ $= \int \frac{1}{x(1+\ln x)}\,dx$ $\quad u = 1+\ln x$

$du = \frac{1}{x}\,dx$

so $\int \frac{1}{u}\,du = \ln|u| + c$

$= \ln|1+\ln x| + C$

2. $\int \frac{\sec x \tan x}{1+\sec^2 x}\,dx$ let $u = \sec x$, $du = \sec x \tan x$

so $\int \frac{1}{1+u^2}\,du = \tan^{-1}(u) + c$

$= \tan^{-1}(\sec x) + C$

$\int u\,dv = uv - \int v\,du$

3. $\int e^{2x} \sin x\;dx$ by parts

$u = e^{2x}$ $\quad v = \sin x$

$du = e^{2x}\,dx$ $\quad dv = -\cos x$

so $\int e^{2x} \sin x \cdot dx = -e^{2x} \cos x + 2\int e^{2x} \sin x\,dx$

again $u = e^{2x}$ $\quad dv = \cos x\,dx$

$du = 2e^{2x}\,dx$ $\quad v = \sin x$

$\int e^{2x} \sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\,dx$

$\Rightarrow \int e^{2x} \sin x\,dx = \frac{e^{2x}}{5}[2\sin x - \cos x] + C$

Partial fraction

4. $\int \frac{3x^2+1}{(x^2-1)(x^2+1)}\, dx$

$$\frac{3x^2+1}{(x^2-1)(x^2+1)}\, dx = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

* $3x^2+1 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x^2-1)$

let $x=1 \rightarrow 4=4A \Rightarrow \boxed{A=1}$

let $x=-1 \rightarrow 4=-4B \Rightarrow \boxed{B=-1}$

by coefficients of

" " of x^3: $0 = A+B+C \Rightarrow \boxed{C=0}$

coefficient of x^2: $3 = A-B+D \rightarrow \boxed{D=-2}$

So

$$\int \frac{3x^2+1}{(x^2-1)(x^2+1)}\, dx = \int \frac{1}{x-1}\, dx - \int \frac{1}{x+1}\, dx - \int \frac{2}{1+x^2}\, dx$$

$$= \ln|x-1| - \ln|x+1| - 2\tan^{-1}x + C$$

$$= \ln\left|\frac{x-1}{x+1}\right| - 2\tan^{-1}x + C$$

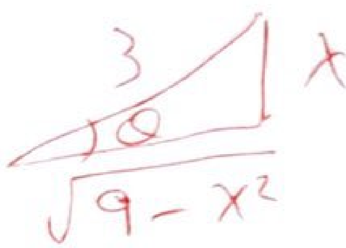
$\frac{1}{x^2\sqrt{9-x^2}}\,dx$ let $x = 3\sin\theta$ $\quad dx = 3\cos x$,

$x^2 = 9\sin^2\theta$

$$= \int \frac{3\cos\theta\, d\theta}{(9\sin^2\theta)(3\cos\theta)} = \frac{1}{9}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{9}\cot\theta + C$$

$$= -\frac{1}{9}\,\frac{\sqrt{9-x^2}}{x} + C$$



6. $\int \frac{1}{(x+3)\sqrt{x^2+6x+13}}\,dx$ Complete sequare.

$x^2+6x+13 = (x+3)^2+4$ $\quad$ let $t = x+3$

$dt = dx$

$\int \frac{dt}{t\sqrt{t^2+4}}$ $\quad$ let $t = 2\tan\theta$, $dt = 2\sec^2\theta\, d\theta$

$\sqrt{t^2+4} = 2\sec\theta$

$$\int \frac{\cancel{2}\sec^{\cancel{2}}\theta}{2\tan\theta\;\cancel{2\sec\theta}}\,d\theta = \frac{1}{2}\int \frac{\sec\theta}{\tan\theta}\,d\theta = \int \csc\theta\, d\theta$$

$$= \frac{1}{2}\ln|\csc\theta + \cot\theta| + C$$

Question 3: (2.5+2+3+3+2.5+2+3)

1. Calculate $\lim_{x\to\infty}(1+\frac{4}{x})^{2x}$.

0.25 0.25 0.25

$$y=\left(1+\frac{4}{x}\right)^{2x} \Rightarrow \ln y = 2x\ln\left(1+\frac{4}{x}\right)$$

$\boxed{\frac{2.5}{2.5}}$

$$\lim_{x\to\infty} 2x\ln\left(1+\frac{4}{x}\right) = \infty \cdot 0 .$$

$$2\lim_{x\to\infty}\frac{\ln\left(1+\frac{4}{x}\right)}{\frac{1}{x}} = \frac{0}{0} \quad 0.25$$

0.25 0.25 0.25

$$2\lim_{x\to\infty}\frac{\frac{1}{1+\frac{4}{x}}\cdot\frac{-4}{x^2}}{\frac{-1}{x^2}} = +8\lim_{x\to\infty}\frac{1}{1+\frac{4}{x}} = +8$$

0.25 0.25

$$\Rightarrow \lim_{x\to\infty} y = e^{+8} \quad 0.25$$

2. Determine whether the improper integral $\int_0^2 \frac{1}{\sqrt{2-x}}\,dx$ converges or diverges.

0.25 0.25

$$\int_0^2 \frac{1}{\sqrt{2-x}}\,dx = \lim_{t\to 2^-}\int_0^t (2-x)^{-\frac{1}{2}}\,dx$$

0.25 0.25

$$= \lim_{t\to 2^-}\left.\frac{(2-x)^{\frac{1}{2}}}{-\frac{1}{2}}\right|_0^t$$

$\boxed{\frac{2}{2}}$

$$= -2\lim_{t\to 2^-}\left((2-t)^{\frac{1}{2}} - (2-0)^{\frac{1}{2}}\right)$$

0.25 0.25

$$= -2\left[0-\sqrt{2}\right]$$

0.25 0.25

$$= 2\sqrt{2}.$$

Converges to $2\sqrt{2}$.

3

$\frac{3}{3}$

3. **Sketch** the region bounded by the graphs of the curves $y = 2 - x^2$ and $y = x$.
Then find its area.

$2 - x^2 = x \Rightarrow x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0$

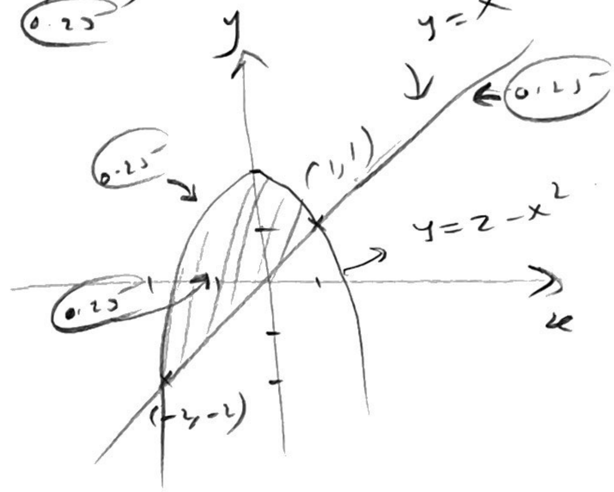
$x = 1$ or $x = -2 \Rightarrow (1, 1), (-2, -2)$

$A = \int_{-2}^{1} (2 - x^2) - x \, dx$

$= \left(2x - \frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_{-2}^{1}$

$= \left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 - \frac{(-2)^3}{3} - \frac{(-2)^2}{2}\right)$

$= \frac{9}{2}$



3 4. **Sketch** the region bounded by the graphs of the curves $y = \frac{x^2}{2}$ and $y = 2$,
then find the volume of the solid generated by revolving this region about the
x –axis.

$\frac{3}{3}$

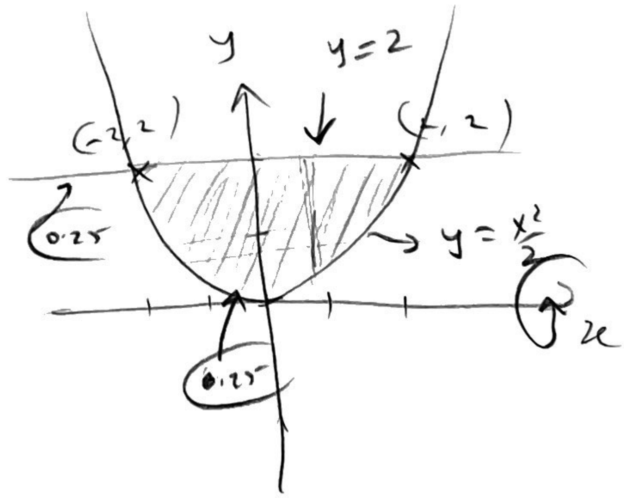
$\frac{x^2}{2} = 2 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2 \Rightarrow (-2, 2), (2, 2)$

$V = 2\pi \int_0^2 (2)^2 - \left(\frac{x^2}{2}\right)^2 dx$

$= 2\pi \int_0^2 4 - \frac{x^4}{4} \, dx$

$= 2\pi \left(4x - \frac{x^5}{20}\right)\Big|_0^2$

$= 2\pi \left(16 - \frac{2^5}{20}\right)$



2.5 5. Find the arc length of $y = \ln(\sec x)$ from $x = 0$ to $x = \frac{\pi}{4}$.

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \left(\frac{d}{dx}(\ln(\sec x))\right)^2}\, dx$$

0.25 0.25 0.25 0.25 0.25 0.25

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \left(\frac{1}{\sec x} \cdot \sec x \tan x\right)^2}\, dx$$

0.25

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\sec^2 x}\, dx$$

0.25

$$= \int_0^{\frac{\pi}{4}} \sec x\, dx$$

0.25

$$= \ln|\sec x + \tan x| \Big|_0^{\frac{\pi}{4}} = \ln\left|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right| - \ln|\sec 0 + \tan 0|$$

0.25

$$= \ln\left|\frac{2}{\sqrt{2}} + 1\right| - \ln|1| = \ln\left|\frac{2}{\sqrt{2}} + 1\right|$$

$\boxed{\frac{2.5}{2.5}}$

2 6. Convert the polar equation $\frac{3}{r} = sin\theta - sec\theta$ into a Cartesian equation.

0.25 0.25

$$3 = r\sin\theta + r\sec\theta \Rightarrow 3 = r\sin\theta - \frac{r}{\cos\theta}$$

0.25

0.25

$$3 = r\sin\theta - \frac{r^2}{r\cos\theta}$$

0.25

0.25

$$3 = y - \frac{x^2+y^2}{x}$$

0.25

0.25

$\boxed{\frac{2}{2}}$

3

7. **Sketch** the region **inside** the curve $r = 1 + sin\theta$ and **outside** the curve $r = 1$, where $0 \le \theta \le \frac{\pi}{2}$, **then find its area**.

3/3

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$r=1+\sin\theta$	1	2	1	0	1

0.25

$$\text{Area} = \int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\sin\theta)^2 - \frac{1}{2}1^2 \, d\theta$$

0.25 0.25 0.25

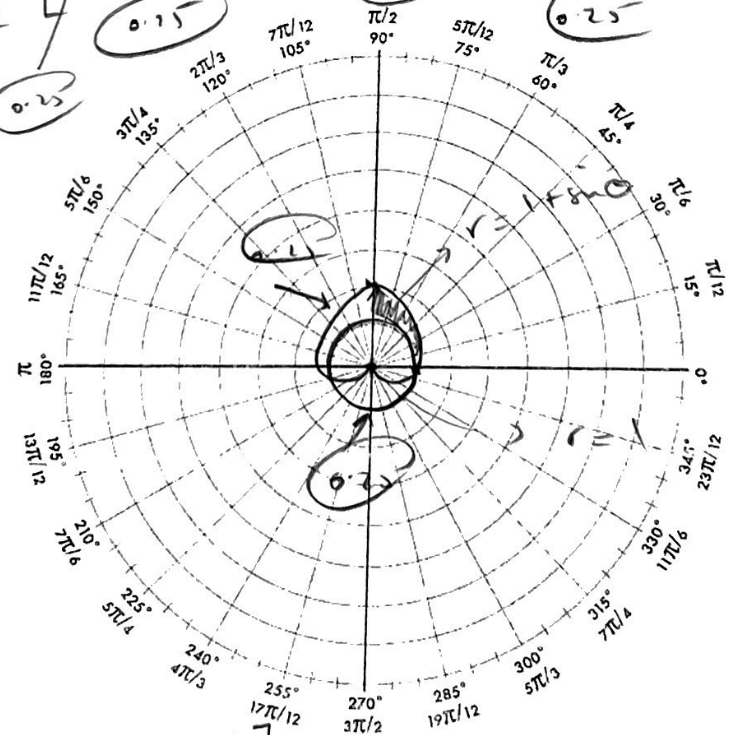
$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \cancel{1} + 2\sin\theta + \sin^2\theta - \cancel{1} \, d\theta$$

0.25 0.25

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin\theta + \frac{1-\cos 2\theta}{2} \, d\theta$$

$$= \frac{1}{2}\left[-2\cos\theta + \frac{1}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)\right]\Big|_0^{\frac{\pi}{2}}$$

0.25 0.25 0.25 0.25



$$= \frac{1}{2}\left[+2 + \frac{1}{2}\left[\frac{\pi}{2}\right]\right] = 1 + \frac{\pi}{8}$$