Question	I	II	III	IV	Total
Mark					

Question I: (1×5=5points)

Question	1	2	3	4	5
Answer					

Choose the correct answer, then fill in the table above:

(1) If $f(x) = \csc^2 x + 6x^2$, then the most general antiderivative is:

(a) $-2\csc^2 x \cot x + 12x$.

(b) $-\cot x + 18x^3 + c$.

(c) $-\cot x + 2\,x^3 + c$.

(d) None of the previous.

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(2) If $\int_a^b [5f(x) + g(x)]dx = 20$ *and* $\int_a^b f(x)dx = 3$, then $\int_a^b g(x)dx =$

(a) 5.

(b) 35.

(c) 17.

(d) None of the previous.

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(3) If $F(x) = \int_{\sin x}^{3} (t^4 + 1)\,dt$, then $F'(x) =$

(a) $\cos x\,(\sin^4 x + 1)$.

(b) $-\cos x\,(\sin^4 x + 1)$.

(c) $4\sin x \cos x\,(\sin^4 x + 1)$.

(d) None of the previous.

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(4) $\int D_x[x\,e^x]dx =$

(a) $e^x(1 + x)$.

(b) xe^x.

(c) $xe^x + c$.

(d) None of the previous.

(5) If $\sum_{k=1}^{4}(k^3 - c) = 60$, then the value of c is

(a) 10.

(b) 40.

(c) -10.

(d) None of the previous.

Question II: (4+3+1.5= 8.5 points)

A. Find the area under the curve $f(x) = x^2 + x$, from 0 to 3, by taking the limit of the Riemann sum and the **right-handed endpoints**. [4 points]

B. Find the value of z that satisfies the conclusion of the Integral Mean Value Theorem for

$f(x) = \frac{1}{x^2}$ on [1,4]. [3 points]

C. Without solving the integral prove that [1.5 points]

$$\int_1^2 \frac{1}{x^2+1}dx \geq \int_1^2 \frac{1}{3x+1}dx\,.$$

Question III: (1.5+2+1.5+2.5= 7.5 points)

A. Evaluate the following integrals:

(i) $\int \frac{\csc^2 t}{\cot^3 t}\, dt$ [1.5 points]

(ii) $\int_0^1 x^3\sqrt{x^4+5}\, dx$ [2 points]

(iii) $\int \sqrt{x}\ (x-1)^2\ dx$ [1.5 points]

B. Use Simpson's rule with $n = 4$ to approximate the integral [2.5 points]

$$\int_1^5 \left(1 - \frac{1}{x}\right) dx.$$

Question IV: (4 points)

A. Sketch the region R bounded by the graphs of the functions $y = \sqrt{x}\ and\ y = 2 - x$ and the $x - axis$. Then **find its area**. [4 points]

Good Luck 😊