

Final Exam - Allowed time: 3 hours
Calculators are not permitted

Q1.

[3] (a) Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & 0 & 1 \end{pmatrix}$. Compute, if possible, $2A + (3B)^t$ and BA .

[2] (b) Compute the determinant $\begin{vmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{vmatrix}$.

[4] (c) Solve by using Gauss-Jordan Elimination Method the linear system

$$\begin{cases} x + y + 2z = 0 \\ 2x + y - z = 3 \\ x - y - 3z = 1 \end{cases}$$

Q2.

[4] (a) Find the standard equation of the ellipse with foci $(5, -2)$ and $(-1, -2)$ and one of its vertices $(7, -2)$ and sketch it.

[4] (b) Find the elements of the conic section of equation $4y^2 - 8y + 16x + 20 = 0$ and then sketch it.

Q3.

[2,3,3] (a) Compute the integrals:

(i) $\int \frac{6x}{(x^2 + 5)^4} dx$, (ii) $\int \frac{1}{x^2} \ln(x^2 + 1) dx$, (iii) $\int \frac{2x + 3}{(x + 1)^2} dx$.

[3] (b) Sketch the region bounded by the curves $y = 4 - x^2$ and $y = x + 2$, and compute its area.

[4] (c) The region bounded by the curves $y = 2 - x^2$ and $y = 2 - x$ is rotated about the y -axis to form a solid \mathcal{S} . Find the volume of \mathcal{S} .

Q4.

[4] (a) The function $y = f(x)$ is defined implicitly by the equation

$$y^4 - xy^2 - e^{-3x} = 0.$$

Compute the derivative $\frac{dy}{dx}$. Evaluate $\frac{dy}{dx}(0)$ knowing that $y(0) = 1$.

[4] (b) Solve the differential equation: $xy' + y = 3x^2$.