King Saud University **Mathematics Department**

Math-254 (NUMERICAL METHODS) Second Semester (1443 H)

TEXTBOOK: **Introduction to Numerical Analysis using MATLAB**

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The students should study the following topics:

Chapter 1: Introduction

Before we start Chapter 2, we must discuss "Computer Representation of Number, **Error**(in details), the Taylor's polynomial. These topic can be found in the recommended book.

Chapter 2: Solution of Nonlinear Equations

Introduction; Shapes of nonlinear equations, How to define it and discuss the conditions under which the root is said to be **simple or multiple**. Discussing four iterative methods for approximating the simple root of nonlinear equations. The bisection method: How to apply it and to compute an error bound for the approximate solutions derived by the method. The fixed point iterative method: How to formulate the function $g(x)$ which will satisfies the conditions of the Theorem 2.4, then apply the iterative scheme and the analysis of the error. The Newton's method: How to apply it and the analysis of its error. The secant method: How to apply it.

Discussing the multiple root of nonlinear equations. Apply one iterative method for approximating the multiple root of nonlinear equations. The approximation of multiple root the following modified Newton's methods should be discussed:

$$x_{n+1} = x_n - \frac{mf(x_n)}{f'(x_n)}, \quad f'(x_n) \neq 0, \quad \text{for} \quad n = 0, 1, 2, \ldots$$

where m is the multiplicity of the multiple roots, and the other one is

$$x_{n+1} = x_n - \frac{f(x_n)f'(x_n)}{[f'(x_n)]^2 - f(x_n)f''(x_n)}, \quad \text{for} \quad n = 0, 1, 2, \ldots$$

The discussion of the rate of convergence of the iterative methods for nonlinear equations

The Newton's method for solving the nonlinear systems (only for two nonlinear equations).

Chapter 3 Systems of Linear Equations

Introduction; linear system and its types, special matrices, direct and iterative methods for solving linear system. How to apply the Gaussian elimination method (without pivoting)(algorithmic

approach) and also, discuss the partial pivoting. Give examples showing that the system has infinite number of solutions or no solution at all (singular matrix).
How to apply LU factorization by using Doolittle's method ($l_{ii} = 1$), and Crout's method ($u_{ii} = 1$). How to apply the iterative methods (Jacobi and Gauss-Seidel) to solve a linear system. The analysis of the error related to these methods (condition for convergence, diagonally dominant matrix ...). Also, how to compute an error bound for both methods.
Error in solving linear systems. Residual vector, condition number of a given matrix... etc. How to compute an upper bound for both absolute and relative errors. Here we must give the definitions of the vector and matrix norms (l_∞ norm only).

Chapter 4: Approximating Functions

Introduction; How to construct the Lagrange polynomial which approximate a function $f(x)$ at an $(n + 1)$ unequally and equally spaced data points. Formulation of nth divided differences of a function. How to apply the divided differences to construct the Newton's divided differences interpolating polynomial. Error in polynomial interpolation: How to compute an error bound for any x value in the interval $[a, b]$ and a given $x = \overline{x} \in [a, b]$. Interpolation using spline functions(Linear Spline Only).

Chapter 5 Differentiation and Integration

Introduction; firstly, we discuss approximation of the derivative of the function at the given point. How to derive the **first** and **second** order finite difference formulas for approximating the first derivative (two and three point formulas) and second derivative (only 3 point formula) of the function $f(x)$ at a point x_0 (note that for the first derivative we study (Forward+Central+Backward) and for the second derivative we study only the central difference formula). How to apply these formulas including the estimation of an error bound.
Secondly, we discuss approximation of the antiderivative (integral) of the function at the given interval. How to derive the Trapezoidal and the Simpson rules, how to apply them and to compute the error bounds (for both single and composite formulas).

Chapter 6: Ordinary Differential Equations

Introduction; Discuss the ordinary differential equations and solve the first order initial-value problems. How to use the Euler's method, the Taylor method of order N and the Runge Kutta method of order **two** (only modified Euler's method) for solving first order initial-value problems in ordinary differential equations. Also, discuss the absolute error.

Note: The Contents of the course will be covered by the following sections:

CHAPTER 1: 1.2,1.3.
CHAPTER 2: 2.1,2.2,2.3,2.4,2.5.
CHAPTER 3: 3.1,3.2,3.3,3.4,3.5,3.6,3.7.
CHAPTER 4: 4.1,4.2,4.3.
CHAPTER 5: 5.1,5.2,5.3,5.4,5.5.
CHAPTER 6: 6.1,6.2,6.3.

Theorems, Lemmas and Notes:
Theorems 2.1-2.6,3.1-3.20,4.1-4.6,5.1-5.5,6.1-6.2.
Lemmas 2.1,2.2,2.3,2.4.
Notes 2.1,2.2,2.3,2.4.

Note: About the **10 Tutorial Marks** we do as follows:
Homework Assignments (2) + **Computer Assignment** (2) + **quiz** (6).
Note: Homework, Computer Assignments and quiz should be taken in the tutorial classes.

NAMES OF CHAPTERS OF THE COURSE

Chapter 1: Introduction to Numerical Methods.
Chapter 2: Solution of Nonlinear Equations. (4 weeks (12 hours))
Chapter 3: Systems of Linear Algebraic Equations. (4 weeks (12 hours))
Chapter 4: Polynomial Interpolation and Approximation. (3 weeks (9 hours))
Chapter 5: Numerical Differentiation and Numerical Integration. (3 weeks (9 hours))
Chapter 6: Numerical Solution of Ordinary Differential Equations. (1week (3 hours))

Computer Assignment: Write computer program of the following method.
Newton's Divided Differences Interpolation Formula for Interpolations. (Chapter 4).

First Short Test: (10 Marks)Tuesday: 28-07-1443(01-03-22) Time:During Lecture.
Second Short Test: (10 Marks)Tuesday: 00-00-1443(00-00-22) Time:During Lecture.
Only One Midterm: (30 Marks)Tuesday: 04-09-1443(05-04-22) Time:9:30-11:30 PM.
Final Exam: (40 Marks)Monday: 29-10-1443(30-05-22) Time:1:0 - 4:00 PM.

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