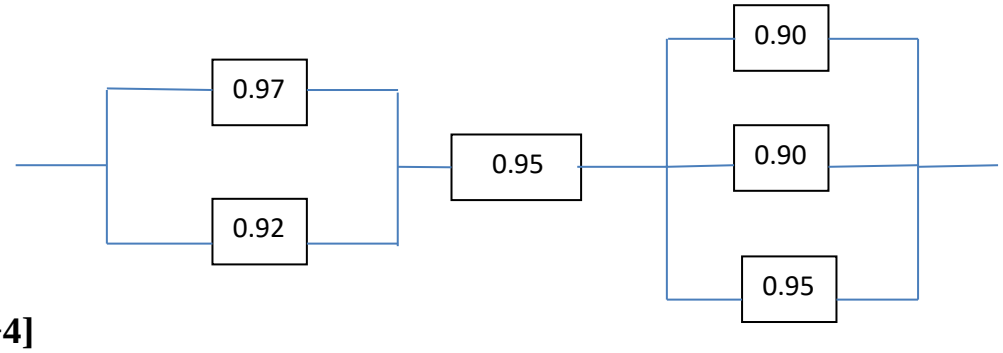
King Saud University
College of Sciences
Department of Mathematics

Mid-Term Exam, S1 1448
M 507 - Advanced Operation Research
Time: 90 Minutes

Answer the following questions (Note that SND Table is attached in page 3).

Q1: [3]

Compute the system reliability for the following configuration diagram where each component has the indicated reliability



Q2: [4+4+4]

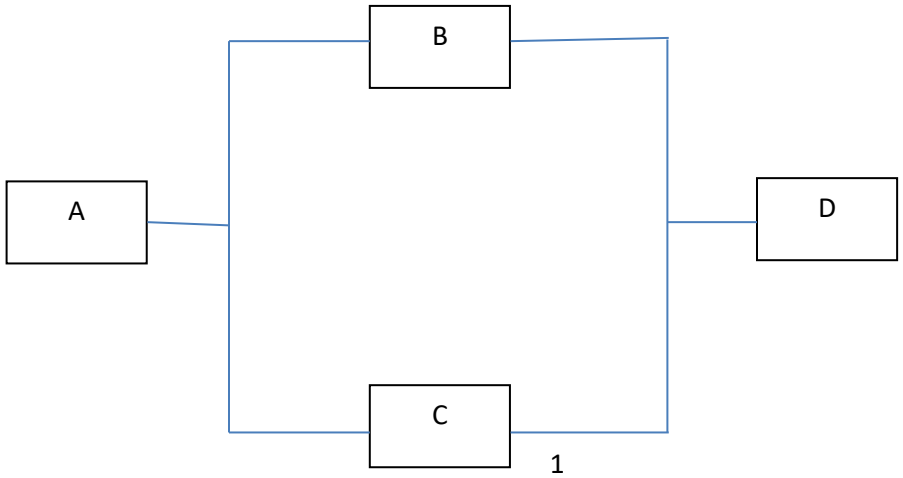
In the reliability diagram below, the reliability of each component is constant and independent. Assuming that each has the same reliability R, compute the system reliability as a function of R using the following methods:

a. Decomposition using B as the keystone element.

b. The reduction method.

c. Compute the importance of each component if $R_A = 0.85$, $R_B = 0.95$,

$R_C = 0.9$ and $R_D = 0.98$



Q3: [5]

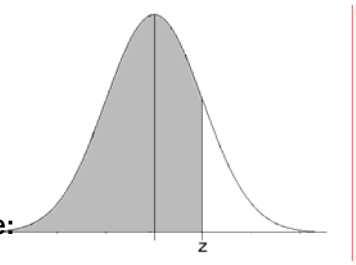
In the reactor safety study, the failure rate random variable is Λ, $\Lambda \sim \text{lognormal}(\upsilon, \tau^2)$ with median $\lambda_m = 6\times10^{-5}$ and a 90% error factor $k = 3$. Find the mean failure rate.

Q4: [5+5]

(a) Show that the failure rate function $z(t)$ at time t can be expressed as $z(t) = \dfrac{1+\mu'(t)}{\mu(t)}$ where $\mu(t) = \text{MRl}(t)$.

(b) Consider an item with mean residual life $\mu(t) = \dfrac{t+2}{t+1}$, where the time t is measured in months. Find $z(t)$, $R(t)$ and MTTF.

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Model Answer

Q1: [3]

$$R_{sys} = [1-(1-0.97)(1-0.92)](0.95)[1-(1-0.9)^2(1-0.95)]$$
$$= 0.94725$$

Q2: [4+4+4]

In the reliability diagram shown in Fig. 1, the reliability of each component is constant and independent. Assuming that each has the same reliability *R*, compute the system reliability as a function of *R* using the following methods:

a) Decomposition using B as the keystone element.

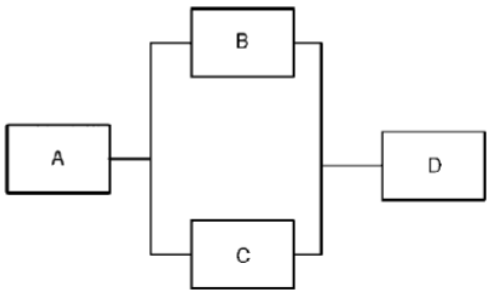


Fig. 1: Reliability diagram

Using B as the keystone element, we have two cases i.e., the case when B functions and the case when it does not.

For the case when B functions, the system reduced to Fig 2.

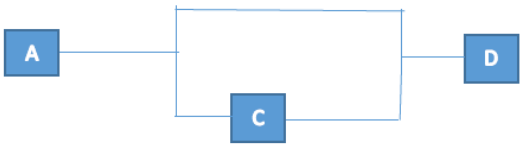


Fig. 2: The case when B functions

Thus the reliability of the system depends only on the reliability of component A and D. Note that $R_A = R_B = R_C = R_D = R$

Therefore,

$$R^+ = R_A R_D = R^2$$

For the case when B fails, the system block is as shown in Fig. 3, which is a series system.

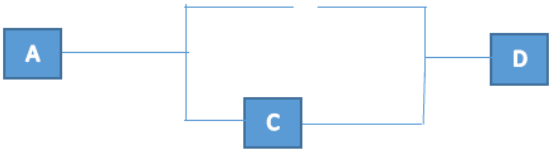


Fig. 3: The case when B fails to work

Thus the reliability of the system depends on A, C, and D, therefore we have:

$$R^- = R_A R_C R_D = R^3$$

Thus the reliability of the system using the two decompositions is given as:

$$R_{system} = R_B R^+ + (1 - R_B) R^-$$
$$R_{system} = R(R^2) + (1 - R) R^3$$
$$R_{system} = 2R^3 - R^4$$

b) Using the reduction method

With this method, it can be seen that components B and C are in parallel and jointly in series with A and D. therefore the reduced system is given in Fig. 4.



Fig. 4: Reduced system

For parallel components B and C, we have

$$R_{B\|C} = 1 - \prod_{i=1}^{2}(1 - R_i)$$
$$R_{B\|C} = R_B + R_C - R_B R_C$$
$$R_{B\|C} = 2R - R^2$$

The reliability of the system is thus given as:

$$R_{system} = R_A R_{B\|C} R_D$$
$$R_{system} = R(2R - R^2)R$$
$$R_{system} = 2R^3 - R^4$$

c)

Recall that the reliability of the system is given as:

$$R_{system} = R_A R_D (R_B + R_C - R_B R_C)$$

The importance of each component is computed by taking the partial derivative with respect to each of the component.

Thus the importance of component A is given as:

$$\frac{\delta R_{system}}{\delta R_A} = \frac{\delta(R_A R_D (R_B + R_C - R_B R_C))}{\delta R_A}$$
$$I_A = R_D (R_B + R_C - R_B R_C)$$

The importance of component B is given as:

$$\frac{\delta R_{system}}{\delta R_B} = \frac{\delta(R_A R_D (R_B + R_C - R_B R_C))}{\delta R_B}$$
$$I_B = R_A R_D - R_A R_D R_C$$

The importance of component C is given as:

$$\frac{\delta R_{system}}{\delta R_C} = \frac{\delta(R_A R_D (R_B + R_C - R_B R_C))}{\delta R_C}$$
$$I_C = R_A R_D - R_A R_B R_D$$

The importance of component D is given as:

$$\frac{\delta R_{system}}{\delta R_D} = \frac{\delta(R_A R_D (R_B + R_C - R_B R_C))}{\delta R_D}$$
$$I_D = R_A (R_B + R_C - R_B R_C)$$

$\Rightarrow I_A = 0.9751,\ I_B = 0.0833,\ I_c = 0.04165$ and $I_D = 0.84575$

Q3: [5]

$\because \Lambda \sim \text{lognormal}(\upsilon, \tau^2)$ where $p(\frac{\lambda_m}{k} < \Lambda < k\lambda_m) = 1 - 2\alpha = 0.90$

The median λ_m and the error factor k are defined respectively as follows.

$\lambda_m = e^{\upsilon}$ and $k = e^{u_\alpha \tau}$, u_α is the upper α% percentile of standard normal distribution (i.e. $\Phi(u_\alpha) = 1 - \alpha$).

at $\alpha = 0.05,\ \Phi(u_\alpha) = 0.95 \Rightarrow u_\alpha = 1.645$ (SND).

$k = e^{u_\alpha \tau} \Rightarrow \tau = \frac{\ln k}{u_\alpha} = \frac{\ln 3}{1.645} \simeq 0.668$

$\lambda_m = e^{\upsilon} \Rightarrow \upsilon = \ln \lambda_m = \ln(6 \times 10^{-5}) = -9.721$

$\therefore \Lambda \sim \text{lognormal}(-9.721, 0.668^2)$

So, the failure rate λ^* is $\lambda^* = e^{\upsilon + \tau^2/2} \simeq 7.5 \times 10^{-5}$ per hour.

Q4: [5+5]

(a)

$$\because \text{MRl}(t) = \int_0^\infty R(x|t)dx = \frac{1}{R(t)} \int_t^\infty R(x)dx$$

$$\therefore \mu(t) = \frac{1}{R(t)} \int_t^\infty R(x)dx \quad (1)$$

$$\therefore \mu'(t) = \frac{1}{R(t)} \left[\frac{d}{dt} \int_t^\infty R(x)dx \right] + \left[\frac{-R'(t)}{R^2(t)} \right] \int_t^\infty R(x)dx \quad (2)$$

$$\because \int_t^\infty R(x)dx = \int_0^\infty R(x)dx - \int_0^t R(x)dx$$

$$\therefore \int_t^\infty R(x)dx = \mu - \int_0^t R(x)dx$$

$$\frac{d}{dt}\int_t^\infty R(x)dx = -\frac{d}{dt}\left[\int_0^t R(x)dx\right] = -R(t) \quad (3)$$

By using (1) and (3) in (2), we obtain

$$\mu'(t) = -1 + \mu(t)z(t) \text{ where } z(t) = \frac{-R'(t)}{R(t)}$$

$$\text{Hence } z(t) = \frac{1+\mu'(t)}{\mu(t)}$$

(b)

$$z(t) = \frac{1+\mu'(t)}{\mu(t)}$$

$$z(t) = \frac{t}{t+1}$$

$$R(t) = \exp(-\int_0^t \frac{u}{u+1}\ du)$$

$$\int_0^t \frac{u}{u+1}\ du = t - \ln(t+1)$$

$$\therefore R(t) = e^{-t+\ln(t+1)} = (t+1)e^{-t}$$

$$\text{MTTF} = \int_0^\infty R(t)dt$$

$$= \int_0^\infty (t+1)e^{-t}dt$$

$$= -te^{-t} - e^{-t} - e^{-t}\Big|_0^{t\to\infty}$$

$$\therefore \text{MTTF} = 0 + 2 = 2 \text{ months}$$

or directly as follows

$$\text{MTTF} = \text{MRl}(0) = 2 \text{ months.}$$
