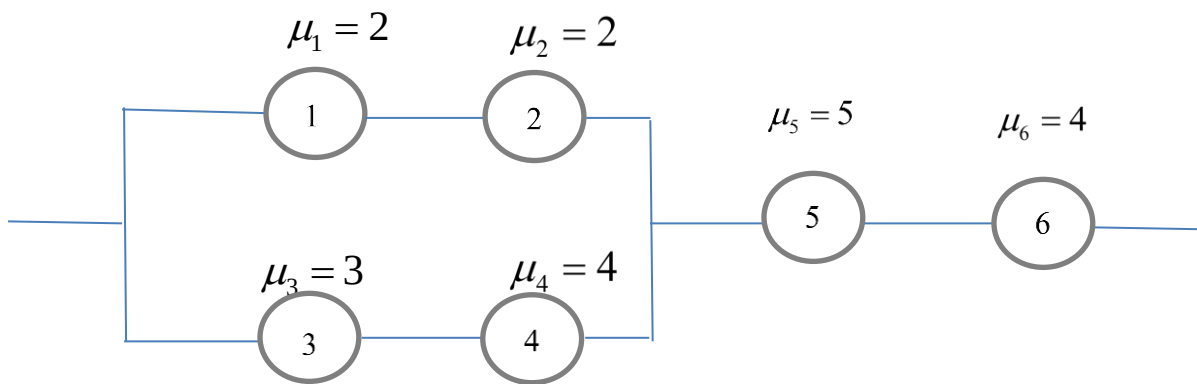




Answer the following questions

Q1: [4+8]

(a) Consider the following system configuration diagram for 6 components, their lifetimes follow exponential distributions with means $\mu_i, i = 1, 2, \dots, 6$ that measured in thousand hours.

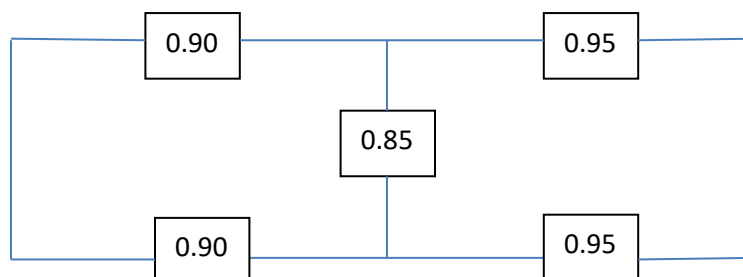


(i) Find the system reliability

(ii) What is the reliability for the system to achieve a life at least 2000 hours?

(b) Compute the system reliability for the following configuration diagram by using the decomposition method and evaluate the importance of component 3 (pivot element in the diagram).

Note that each component in the diagram has the indicated reliability.



Q2: [6+6]

(a) A rotary pump has a constant failure rate $\lambda = 4.28 \times 10^{-4}$ hours⁻¹.

(i) Determine the mean time to failure (MTTF) of the pump and find the probability that it survives the first month ($t_1 = 730$ hours).

(ii) Suppose that the pump has been functioning without failure during its first month, find the probability that the pump will fail during the next month ($t_2 = 730$ hours).

(b) Consider an item with failure rate function $z(t) = \frac{t}{t+1}$. The failure rate function is increasing and approaches 1 where $t \rightarrow \infty$. Find $R(t)$, MTTF and the mean residual life $MRL(t)$.

Q3: [6]

The time to failure T of a variable choke valve is assumed to have a Weibull distribution with shape parameter $\alpha = 2$ and scale parameter $\lambda = 1.15 \times 10^{-4}$ per hour. Determine each of the following.

(a) The probability that the valve will survive 6 months ($t = 4380$ hours) in continuous operation.

(b) The mean time to failure.

(c) The median life.

Model Answer

Q1: [4+8]

(a)

(i)

For parallel components, $R_{\text{sys}} = 1 - \prod_{i=1}^n (1 - R_i)$

For series components, $R_{\text{sys}} = \prod_{i=1}^n R_i$

For the given diagram,

$$\because R_{1234}(t) = 1 - [1 - R_{12}(t)][1 - R_{34}(t)]$$

$$\text{and } \because R_{\text{sym}}(t) = R_{1234}(t)R_5(t)R_6(t)$$

$$\therefore R_{\text{sym}}(t) = [1 - (1 - e^{-t})(1 - e^{-7t/12})]e^{-9t/20}$$

(ii)

$$R_{\text{sym}}(2) = [1 - (1 - e^{-2})(1 - e^{-7/6})]e^{-0.9} \\ \approx 0.1645$$

(b)

We use the decomposition method and we take the component 3 of reliability 0,95 as a pivot element.

$$R^+ = [1 - (1 - 0.9)^2][1 - (1 - 0.95)^2] \\ = 0.987525$$

$$R^- = 1 - (1 - 0.9 \times 0.95)(1 - 0.9 \times 0.95) \\ = 0.978975$$

$$\therefore R_{\text{sys}} = R_3 R^+ + (1 - R_3) R^- \\ = 0.85 \times 0.987525 + 0.15 \times 0.978975 \\ \approx 0.986 = 98.6\%$$

The importance of component 3 (pivot element in the diagram) is

$$R^+ - R^- = 8.55 \times 10^{-3}$$

Q2: [6+6]

(a)

(i) The mean time to failure of the pump is

$$\text{MTTF} = \frac{1}{\lambda} = \frac{1}{4.28 \times 10^{-4}}$$

$$\approx 2336 \text{ hours} \approx 3.2 \text{ months.}$$

The probability that the pump survives the first month ($t_1 = 730$ hours) is

$$R(t) = e^{-\lambda t} = e^{-4.28 \times 10^{-4} (730)}$$

$$\approx 0.732 = 73.2\%.$$

(ii) The probability that the pump will fail during the next month ($t_2 = 730$ hours), knowing that the pump has been functioning without failure during its first month is

$$\Pr(T \leq t_1 + t_2 | T > t_1) = \Pr(T \leq t_2)$$

$$= 1 - e^{-4.28 \times 10^{-4} (730)} \approx 0.268 = 26.8\%,$$

since the pump is as good as new when it is still functioning at time t_1 (memoryless property of exponential distribution).

(b)

The reliability (survival) function is

$$R(t) = \exp\left(-\int_0^t z(u)du\right)$$

$$R(t) = \exp\left(-\int_0^t \frac{u}{u+1} du\right)$$

$$\because \int_0^t \frac{u}{u+1} du = t - \ln(t+1)$$

$$\therefore R(t) = (t+1)e^{-t}$$

$$\text{MTTF} = \int_0^{\infty} R(t)dt$$

$$\begin{aligned}\text{MTTF} &= \int_0^{\infty} (t+1)e^{-t} dt \\ &= -te^{-t} - e^{-t} - e^{-t} \Big|_0^{t \rightarrow \infty}\end{aligned}$$

$$\therefore \text{MTTF} = 2 \text{ months.}$$

The conditional survival function is

$$\begin{aligned}\because R(x|t) &= \frac{R(x+t)}{R(t)} \\ &= \frac{(t+x+1)e^{-(t+x)}}{(t+1)e^{-t}}\end{aligned}$$

$$\because R(x|t) = \frac{t+x+1}{t+1} e^{-x}$$

The mean residual life is

$$\begin{aligned}\because \text{MRL}(t) &= \int_0^{\infty} R(x|t)dx \\ &= \int_0^{\infty} \left(1 + \frac{x}{t+1}\right) e^{-x} dx\end{aligned}$$

$$\text{MRL}(t) = -e^{-x} \Big|_0^{x \rightarrow \infty} + \frac{1}{t+1} \left[-xe^{-x} - e^{-x} \right]_0^{x \rightarrow \infty}$$

$$\therefore \text{MRL}(t) = 1 + \frac{1}{t+1}$$

Note that $\text{MRL}(0) = \text{MTTF} = 2 \text{ months.}$

Q3: [6]

(a)

$$\begin{aligned}\because R(t) &= e^{-(\lambda t)^\alpha} \text{ for } T \sim \text{Weibull}(\alpha, \lambda) \\ &= e^{-(1.15 \times 10^{-4} \times 4380)^2}\end{aligned}$$

$$\therefore R(4380) \approx 0.78$$

(b)

$$\begin{aligned}\text{MTTF} &= \frac{1}{\lambda} \Gamma\left(\frac{1}{\alpha} + 1\right) = \frac{\Gamma(1.5)}{1.15 \times 10^{-4}} \\ &= \frac{0.5\Gamma(0.5)}{1.15 \times 10^{-4}} = \frac{0.5\sqrt{\pi}}{1.15 \times 10^{-4}} \\ &\approx 7706.3 \text{ hours.}\end{aligned}$$

(c)

$$\begin{aligned}t_m &= \frac{1}{\lambda} (\ln 2)^{1/\alpha} \\ &= \frac{1}{1.15 \times 10^{-4}} (\ln 2)^{0.5} \approx 7239.6 \text{ hours}\end{aligned}$$
