King Saud University
College of Sciences
Department of Mathematics

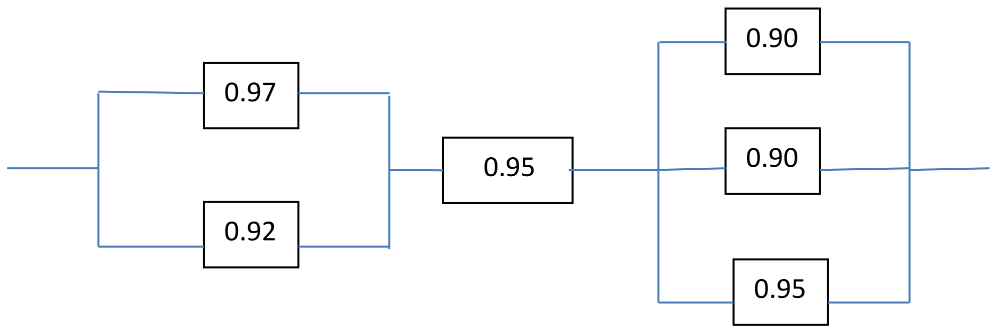
Final Exam, S1 1443
M 507 - Advanced Operations Research
Time: 3 hours – **Marks: 40**

Answer the following questions.

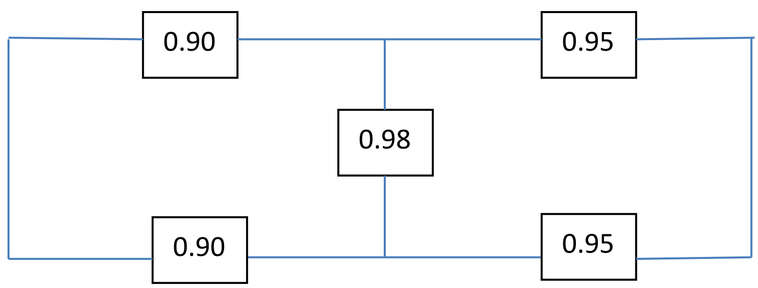
Q1: [4+4]

Compute the system reliability for the following configuration diagram where each component has the indicated reliability

(a)



(b)



Q2: [4+4]

(a) Using the differential equations

$$\frac{dp_0(t)}{dt} = -\lambda p_0(t) \qquad (1)$$

$$\frac{dp_n(t)}{dt} = \lambda p_{n-1}(t) - \lambda p_n(t), \;\; n = 1, 2, 3, \; \ldots \qquad (2)$$

where all birth parameters are the same constant λ with initial condition $X(0)=0$, Show that $p_n(t) = \dfrac{(\lambda t)^n e^{-\lambda t}}{n!}$, $n = 0,1,2,...$

(b) Messages arrive at a telegraph office as a Poisson Process with mean rate of 3 messages per hour.

(i) What is the probability that no messages arrive during the morning hours 8:00 A.M. to noon?

(ii) What is the distribution of the time at which the first afternoon message arrives?

Q3: [4+4]

If X is the life of an item of a product. Find the mean time to failure MTTF, the median $x_{0.50}$, the failure rate at 500 hours $\lambda(500)$, and also, determine the probability that the item will survive until age 500 hours, in each of the following cases.

(a) $X \sim \text{Weibull}(\eta, \beta)$ where $\beta = 1.5,\ \eta = 1000$

(b) $X \sim \text{Lognormal}(\mu, \sigma^2)$ where $\mu = 6.908,\ \sigma = 0.317$

Q4: [4+4]

(a) For the Markov process $\{X_t\}$, $t = 0,1,2,...,n$ with states $i_0, i_1, i_2, \ldots, i_{n-1}, i_n$

Prove that: $\Pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \ldots, X_n = i_n\} = p_{i_0} P_{i_0 i_1} P_{i_1 i_2} \ldots P_{i_{n-1} i_n}$ where $p_{i_0} = \Pr\{X_0 = i_0\}$

(b) The random variables $\xi_1, \xi_2, \ldots$ are independent and with the common probability mass function

$k =$	0	1	2	3
$\Pr\{\xi = k\} =$	0.1	0.3	0.2	0.4

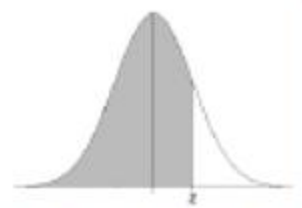
Set $X_0 = 0$, and let $X_n = \max\{\xi_1, \ldots, \xi_n\}$ be the largest ξ observed to date. Determine the transition probability matrix for the Markov chain $\{X_n\}$.

Q5: [8]

A company produces two types of mobile phones, Model A and Model B and that it takes 5 hours to produce a unit of Model A and two hours to produce a unit of Model B, knowing that the number of working hours is 900 hours per week. Consider that the unit cost of Model A is $8, and the unit cost of Model B is $10 and the budget for production per week is $2800. If the profit per unit of Model A is $3 and the profit per unit of Model B is $2, how many mobiles of each model are needed to produce per week to make the maximum profit?

Table 1

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table 2

$B_2 - B_1^2 = \Gamma(2/\beta + 1) - \Gamma^2(1/\beta + 1)$ as a Function of the Shape Parameter β

β	B_1	$B_2 - B_1^2$
1.0	1.0000	1.0000
1.1	0.9649	0.7714
1.2	0.9407	0.6197
1.3	0.9336	0.5133
1.4	0.9114	0.4351
1.5	0.9027	0.3757
1.6	0.8966	0.3292
1.7	0.8922	0.2919
1.8	0.8893	0.2614
1.9	0.8874	0.2360
2.0	0.8862	0.2146
2.5	0.8873	0.1441
3.0	0.8930	0.1053
3.5	0.8997	0.0811
4.0	0.9064	0.0647
5.0	0.9182	0.0442

Model Answer

Q1: [4+4]

(a)

$$R_{sys} = [1-(1-0.97)(1-0.92)](0.95)[1-(1-0.9)^2(1-0.95)]$$
$$= 0.94725$$

(b)

We use the decomposition method and we take the component 3 of reliability 0.98 as a pivot element.

$$R^+ = [1-(1-0.9)^2][1-(1-0.95)^2]$$
$$= 0.987525$$
$$R^- = 1-(1-0.9\times 0.95)(1-0.9\times 0.95)$$
$$= 0.978975$$
$$\therefore\ R_{sys} = R_3R^+ + (1-R_3)R^-$$
$$= 0.98\times 0.987525 + 0.02\times 0.978975$$
$$= 0.9874$$

Q2: [4+4]

(a)

$$\frac{dp_0(t)}{dt} = -\lambda p_0(t) \qquad (1)$$
$$\frac{dp_n(t)}{dt} = \lambda p_{n-1}(t) - \lambda p_n(t),\ n = 1, 2, 3,\ \ldots \qquad (2)$$

Let $X(t)$ represents the size of the population, and the initial condition is

$$X(0) = 0 \quad \Rightarrow\ p_0(0) = 1$$

$$\Rightarrow\ p_n(0) = \begin{cases} 1 & ,\ n = 0 \\ 0 & ,\ \text{otherwise} \end{cases}$$

$$(1) \Rightarrow \frac{dp_0(t)}{dt} = -\lambda p_0(t)$$

$$\therefore \quad \frac{dp_0(t)}{p_0(t)} = -\lambda dt$$

$$\int_0^t \frac{dp_0(u)}{p_0(u)} = -\lambda \int_0^t du$$

$$\left[\ln p_0(u)\right]_0^t = -\lambda t$$

$$\ln p_0(t) - \ln p_0(0) = -\lambda t$$

$$\ln p_0(t) - \ln 1 = -\lambda t \text{ , where } p_0(0) = 1$$

$$\therefore \quad \ln p_0(t) = -\lambda t \;\Rightarrow p_0(t) = e^{-\lambda t} \qquad (3)$$

$$(2) \Rightarrow \frac{dp_n(t)}{dt} = p_{n-1}(t) - \lambda p_n(t), \; n = 1, 2, 3, \; ...$$

$$\therefore \quad \frac{dp_n(t)}{dt} + \lambda p_n(t) = \lambda p_{n-1}(t)$$

Multiply both sides by $\mathrm{e}^{\lambda t}$

$$\mathrm{e}^{\lambda t}\left[\frac{dp_n(t)}{dt} + \lambda p_n(t)\right] = \lambda p_{n-1}(t)\mathrm{e}^{\lambda t}$$

$$\therefore \quad \frac{d}{dt}\left[e^{\lambda t} p_n(t)\right] = \lambda p_{n-1}(t)\mathrm{e}^{\lambda t}$$

$\therefore$ By separation of variables and Integration from 0 to t, we get

$$\int_0^t d\left[e^{\lambda x} p_n(x)\right] = \lambda \int_0^t p_{n-1}(x)\mathrm{e}^{\lambda x} dx$$

$$\left[e^{\lambda x} p_n(x)\right]_0^t = \lambda \int_0^t p_{n-1}(x)\mathrm{e}^{\lambda x} dx$$

$$e^{\lambda t} p_n(t) - p_n(0) = \lambda \int_0^t p_{n-1}(x)\mathrm{e}^{\lambda x} dx, \; n = 1, 2, 3, \; ...$$

$$p_n(t) = \lambda \mathrm{e}^{-\lambda t} \int_0^t p_{n-1}(x)\mathrm{e}^{\lambda x} dx, \; n = 1, 2, 3, \; ... \qquad (4)$$

which is a recurrence relation

$at\ n=1$

$$(4)\Rightarrow p_1(t)=\lambda e^{-\lambda t}\int_0^t p_0(x)e^{\lambda x}dx$$

$\because p_0(x)=e^{-\lambda x}$ from eq. (3)

$$\therefore p_1(t)=\lambda e^{-\lambda t}\int_0^t e^{-\lambda x}e^{\lambda x}dx$$

$$=\lambda e^{-\lambda t}\int_0^t dx$$

$$\therefore p_1(t)=\lambda t e^{-\lambda t} \qquad (5)$$

$at\ n=2$

$$(4)\Rightarrow p_2(t)=\lambda e^{-\lambda t}\int_0^t p_1(x)e^{\lambda x}dx$$

$\because p_1(x)=\lambda x e^{-\lambda x}$ from eq. (5)

$$\therefore p_2(t)=\lambda e^{-\lambda t}\int_0^t \lambda x e^{-\lambda x}e^{\lambda x}dx$$

$$=\lambda^2 e^{-\lambda t}\int_0^t x dx$$

$$\therefore p_2(t)=\lambda^2 e^{-\lambda t}\left[\frac{x^2}{2}\right]_0^t$$

$$\therefore p_2(t)=\frac{(\lambda t)^2 e^{-\lambda t}}{2!} \qquad (6)$$

From Eqs (3), (5) and (6), we can deduce that $p_n(t)=\dfrac{(\lambda t)^n e^{-\lambda t}}{n!},\ n=0,1,2,\ldots$

(b)

(i) For Poisson Process $\{X(t);\ t\geq 0\}$, where $X(t)$ is the random variable that represents the number of messages that arrive at the telegraph office at any time t.

$$\Pr\{X(s+t)-X(s)=k\}=\frac{(\lambda t)^k e^{-\lambda t}}{k!},\ k=0,1,2,\ldots$$

$$\therefore \Pr\{X(12)-X(8)=0\}=\frac{(3\times 4)^0 e^{-3(4)}}{0!}=e^{-12}$$

$$\approx 6.1442\times 10^{-6}$$

where $\lambda = 3,\ t = 12 - 8 = 4$ and $k = 0$

(ii) Consider T is the random variable that represents the time at which the first afternoon message arrives. Afternoon is the period between 12:00 P.M. and 12:00 A.M. i.e. $t \in (12, 24)$
So, we can write

$$\Pr(T > t) = \Pr\{\text{The first afternoon message arrives after t units of time}\}$$
$$= \Pr\left\{X(t) - X(12) = 0\right\}$$
$$= \frac{[3(t-12)]^0 e^{-3(t-12)}}{0!}$$
$$= e^{-3(t-12)}$$

Which is the survival/reliability function.
Also,

$$\Pr(T \le t) = 1 - \Pr(T > t)$$
$$= 1 - e^{-3(t-12)}$$

$\therefore\ \Pr(T \le t) = 1 - e^{-3x}$, where $x = t - 12$

which is the cumulative distribution function.

$\therefore\ T \sim \exp(3)$

i.e. $T \sim$ exponential distribution with parameter equals 3.

Q3: [4+4]

(a)

For Weibull distribution

MTTF

$$MTTF = \eta\ \Gamma\left(\frac{1}{\beta} + 1\right)$$
$$= 1000\ \Gamma\left(\frac{1}{1.5} + 1\right)$$
$$= \eta\ B_1$$
$$= 1000 \times 0.9027$$
$$= 902.7$$

The median $x_{0.50}$

$$x_p = \left(\ln\left(\frac{1}{1-p} \right) \right)^{1/\beta} \times \eta$$

$$x_{0.50} = \left(\ln\left(\frac{1}{1-0.50} \right) \right)^{1/1.5} \times 1000$$

$$= (\ln 2)^{1/1.5} \times 1000$$

$$= 783.2198$$

The failure rate at 500 hours $\lambda(500)$,

$$\lambda(x) = \frac{f(x)}{R(x)}$$

$$= \frac{\frac{\beta}{\eta}\left[\frac{x}{\eta}\right]^{\beta-1} \exp[-(\frac{x}{\eta})^{\beta}]}{\exp[-(\frac{x}{\eta})^{\beta}]}$$

$$= \frac{\beta}{\eta}\left[\frac{x}{\eta}\right]^{\beta-1}$$

$$\lambda(500) = \frac{1.5}{1000} \times \left[\frac{500}{1000}\right]^{1.5-1}$$

$$= \frac{1.5}{1000}(0.5)^{0.5}$$

$$= 1.0607 \times 10^{-3} \; hours$$

The reliability at 500 hours $R(500)$,

$$R(x) = \exp[-(\tfrac{x}{\eta})^{\beta}]$$

$$= e^{-(\frac{500}{1000})^{1.5}}$$

$$= \exp\left[-(0.5)^{1.5}\right]$$

$$= 0.70219$$

(b)

For Lognormal distribution

MTTF

$$MTTF = e^{\mu + \sigma^2/2}$$

$$= e^{[6.908 + 0.5(0.317)^2]}$$

$$= 1051.785526$$

The median $x_{0.50}$

$$x_p = \exp(\mu + z_p \sigma)$$
$$x_{0.50} = \exp(\mu + 0 \times \sigma)$$
$$= e^{\mu}$$
$$= e^{6.908}$$
$$= 1000.244751$$

The failure rate at 500 hours $\lambda(500)$,

$$\lambda(x) = \frac{f(x)}{R(x)}$$
$$= \frac{\frac{1}{\sigma x}\varphi\left[\frac{\ln x - \mu}{\sigma}\right]}{1 - \Phi\left[\frac{\ln x - \mu}{\sigma}\right]}$$
$$= \frac{\frac{1}{\sigma x}\frac{1}{\sqrt{2\pi}\sigma}e^{-(\ln x - \mu)^2/2\sigma^2}}{1 - \Phi\left[\frac{\ln x - \mu}{\sigma}\right]}$$
$$\lambda(500) = \frac{7.258965119 \times 10^{-4}}{\Phi(2.19)}$$
$$= \frac{7.258965119 \times 10^{-4}}{0.9857}$$
$$= 7.3643 \times 10^{-4} \text{ hours}$$

The reliability at 500 hours $R(500)$,

$$R(x) = 1 - \Phi\left[\frac{\ln x - \mu}{\sigma}\right]$$
$$= 1 - \Phi\left[\frac{\ln 500 - 6.908}{0.317}\right]$$
$$= 1 - \Phi[-2.19]$$
$$= \Phi(2.19)$$
$$\therefore R(500) = 0.9857$$

Q4: [4+4]

(a)

$$\because \Pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n\}$$
$$= \Pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\}.\Pr\{X_n = i_n | X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\}$$
$$= \Pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\}.P_{i_{n-1}i_n}$$ Definition of Markov

By repeating this argument $n-1$ times

$$\therefore \Pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n\}$$
$= p_{i_0} P_{i_0 i_1} P_{i_1 i_2} \dots P_{i_{n-2} i_{n-1}} P_{i_{n-1} i_n}$ where $p_{i_0} = \Pr\{X_0 = i_0\}$ is obtained from the initial distribution of the process.

(b)

For transition probability matrix of a Markov chain

The elements of first row are given by

$$P_{0,0} = \Pr\{X_1 = 0\} = p_0 = 0.1$$
$$P_{0,1} = \Pr\{X_1 = 1\} = p_1 = 0.3$$
$$P_{0,2} = \Pr\{X_1 = 2\} = p_2 = 0.2$$
$$P_{0,3} = \Pr\{X_1 = 3\} = p_3 = 0.4$$

The elements of second row are given by

$P_{1,0} = 0$ where X_n cannot decrease

$$P_{1,1} = \Pr\{X_n = 1 | X_{n-1} = 1\} = \Pr\{\xi \le 1\} = 0.1 + 0.3 = 0.4$$
$$P_{1,2} = \Pr\{X_n = 2 | X_{n-1} = 1\} = \Pr\{\xi = 2\} = 0.2$$
$$P_{1,3} = \Pr\{X_n = 3 | X_{n-1} = 1\} = \Pr\{\xi = 3\} = 0.4$$

The elements of third row are given by

$P_{2,0} = P_{2,1} = 0$ where X_n cannot decrease

$$P_{2,2} = \Pr\{X_n = 2 | X_{n-1} = 2\} = \Pr\{\xi \le 2\} = 0.1 + 0.3 + 0.2 = 0.6$$
$$P_{2,3} = \Pr\{X_n = 3 | X_{n-1} = 2\} = \Pr\{\xi = 3\} = 0.4$$

The elements of fourth row are given by

$P_{3,0} = P_{3,1} = P_{3,2} = 0$ where X_n cannot decrease

$$P_{3,3} = \Pr\{X_n = 3 | X_{n-1} = 3\} = \Pr\{\xi \le 3\} = 0.1 + 0.3 + 0.2 + 0.4 = 1$$

The transition probability matrix will be of the form

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \left\| \begin{array}{cccc} 0.1 & 0.3 & 0.2 & 0.4 \\ 0 & 0.4 & 0.2 & 0.4 \\ 0 & 0 & 0.6 & 0.4 \\ 0 & 0 & 0 & 1 \end{array} \right\| \end{array}$$

Q5: [8]

The LP pb. is of form:

$$\begin{aligned} \max \quad & 3x_1 + 2x_2 \\ & 5x_1 + 2x_2 \le 900 \\ & 8x_1 + 10x_2 \le 2800 \quad \Rightarrow \\ & x_1 \ge 0 \;\; , \;\; x_2 \ge 0 \end{aligned}$$

The canonical form:

$$\begin{aligned} \max \quad & 3x_1 + 2x_2 \\ & 5x_1 + 2x_2 + x_3 = 900 \\ & 8x_1 + 10x_2 + x_4 = 2800 \\ & x_1 \ge 0 \;\; , \;\; x_2 \ge 0 \end{aligned}$$

where x_3 and x_4 are slack variables.

Let $x_1 = x_2 = 0 \quad \Rightarrow$ NBVs = $\{x_1, x_2\}$ and BVs = $\{x_3, x_4\}$

$$\Rightarrow \quad \begin{aligned} x_3 &= 900 - 5x_1 - 2x_2 \\ x_4 &= 2800 - 8x_1 - 10x_2 \\ z &= 0 + 3x_1 + 2x_2 \end{aligned}$$

1st dictionary

Let x_1 be incoming variable (it has a +ve coefficient In the equation for z)

Ratio test

$x_3 : \frac{900}{5} = 180, \quad x_4 : \frac{2800}{8} = 350$

Min. ratio for x_3

$\therefore\ x_3 \rightarrow$ outgoing variable

$$\Rightarrow\ x_1 = 180 - 2/5\ x_2 - 1/5\ x_3$$
$$x_4 = 1360 - 34/5\ x_2 + 8/5\ x_3$$

$$x_1 = 180 - 2/5\ x_2 - 1/5\ x_3$$
$$\Rightarrow\ x_4 = 1360 - 34/5\ x_2 + 8/5\ x_3$$
$$z = 540 + 4/5\ x_2 - 3/5\ x_3$$

2nd dictionary

Let x_2 be incoming variable (it has a +ve coefficient In the equation for z)

Ratio test

$$x_1 : \frac{180}{2/5} = 450, \quad x_4 : \frac{1360}{34/5} = 200$$

Min. ratio for x_4

$\therefore\ x_4 \rightarrow$ outgoing variable

$$\Rightarrow\ x_2 = 200 + 4/17\ x_3 - 5/34\ x_4$$
$$x_1 = 180 - \frac{2}{5}(200 + 4/17\ x_3 - 5/34\ x_4) - 1/5 x_3$$

$$x_1 = 100 - 5/17\ x_3 + 5/85\ x_4$$
$$\Rightarrow\ x_2 = 200 + 4/17\ x_3 - 5/34\ x_4$$
$$z = 700 - 7/17\ x_3 - 2/17\ x_4$$

3rd dictionary

Here, we have –ve coefficients for all variables in the z equation, so we should stop.

$\therefore$ The optimal solution is $x_1 = 100,\ x_2 = 200$ where max $z = \$700$